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A self-organized model of technological progress

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We develop a scenario for technological evolution where a finite number of agents interact locally in a low dimensional space. Agents adopt a "technology" and obtain a payoff that depends both on how advanced is this technology and how "compatible" it is with those of her neighbors/partners. Progress is induced through stochastic innovation and inter-agent coordination. We show that under these conditions the system displays a rich variety of behaviour ranging from synchronization (technology grows as a common front) to criticality where advance is elicited in terms of "technological avalanches" distributed according to a power law. We also show that is it possible to define a macroscopic observable (rate of technological progress) which is maximized in the critical region.

I. INTRODUCTION

In modern economies, technological change is a rapid phenomenon that has an inherent systemic nature to it. Given the crucial complementarities involved in using advanced technology, economic agents will only find it profitable to advance along the ladder open by new technological possibilities if others also follow a similar path [1–4].

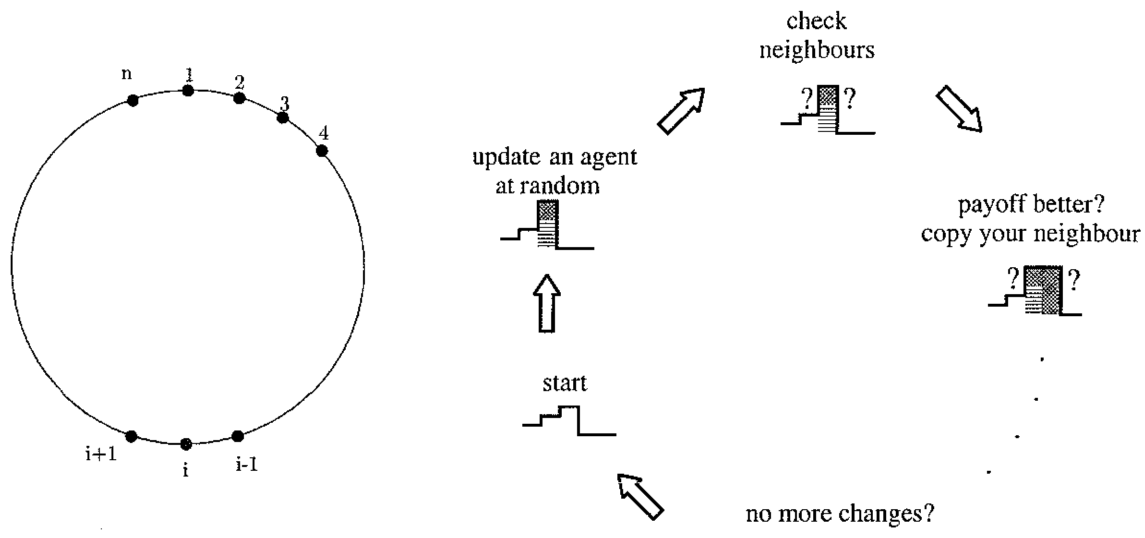
This would seem to indicate that any substantial "technological update" to be undertaken in a large economy will require some sort of implicit or explicit coordination (i.e. "good timing") among a large number of agents. For only then will these agents reap the benefits (and thus have the incentives to incur the costs) of a more sophisticated technology and the transition may indeed take place. This raises the important question of how is it that large and decentralized societies develop suitable mechanisms that render such coordination feasible. A natural way out of this puzzle is given by the observation that, typically, agents do not interact with the population at large but only with a small subset of "neighbors". Therefore, it is conceivable that a technological transition may occur in gradual steps, in each of these only a certain string of neighboring agents managing to internalize the required technological complementarities.

If indeed technological transitions take place in this manner, one should observe the materialization and persistence of a substantial degree of heterogeneity across an economy: some "islands" are rather advanced, others stay relatively backward, still others display an intermediate range of development and act as bridges between the former two. Under some circumstances, these technological bridges may be rather robust. In others, however, they will be rather fragile, thus triggering technological avalanches when slightly disturbed. In each case, the aggregate technological dynamics of the economy will be very different.

In this paper, we want to construct a model of technological advance with the characteristics outlined. Agents interact locally along a certain (one-dimensional) lattice. Most of the time, their technological decisions are taken as a best response to the configuration prevailing in their neighborhoods. Occasionally, however, the economy is perturbed, some randomly chosen agent performing a "unilateral innovation" (i.e. an exogenous upward shift in her technological choice). We are interested in studying the aggregate long-run behavior generated by this process. In particular, we want to identify long-run regularities on the distribution of "technological avalanches", here understood as the waves of technological advance that follow a single perturbation of the economy as indicated.

As we shall see, these avalanches exhibit the long-run regularities displayed by self-organized critical systems, of the sort extensively studied in the physics literature in the last decade (e.g. [5–7]). Once this is confirmed, our main ensuing concern will be as follows.

First, we shall identify the parameter region where criticality arises. As it turns out, this phenomenon will be seen to materialize whenever the incompatibility (or miscoordination) costs exceed a certain threshold.



(a) Distribution of the agents in a one-dimensional ring.

(b) Schematic dynamics of the model.

FIG. 1: Description of the investigated model.

Second, we address the issue of "endogenizing" incompatibility costs. Of course, there are some contexts where these costs are best conceived as exogenous, i.e. a datum of the model. However, in some other cases, one may think of them as being the outcome of some underlying process of adjustment or the target of economic policy. In this latter case, we may ask: What is the level for such costs that maximizes the rate of technological advance of the system? Quite strikingly, we find that the advance of the system is maximized when incompatibility costs are close to the lower boundary of (and within) the critical region. Using a well-known phrase of Kauffman [8], we could describe matters as suggesting that the performance of the system is optimized when the process is "at the edge of order and chaos". Resorting to convenient approximations, this conclusion will be analytically understood as the achievement of a suitable balance between the "hold" induced by large incompatibility costs in maintaining some (beneficial) degree of population heterogeneity and the "pull" of innovation toward homogenization (i.e. technological diffusion) that is favored by low such costs.

Finally, we attempt to use our model to shed some light on real-world phenomena embodying indefinite advance of "knowledge", technological or otherwise. In the future, our research in this respect will aim at shedding some light on the technological dynamics of some of the most innovating and pervasive sectors of modern economies, e.g. the computer industry. For the moment, we are just in the position to establish some preliminary and merely qualitative parallelisms with the advance of "academic knowledge", as indirectly captured by available data on scientific citations. As we shall explain, some of this empirical evidence concerning this phenomenon indicates the existence of self-organized critical behavior.

II. THE FRAMEWORK

We consider n agents, each of them occupying a particular node in a one-dimensional boundariless lattice (cf. Fig. 1(a)). Time is discrete. At every $t = 0, 1, 2, ...$, each agent $i \in \{1, 2, ..., n\}$ adopts a certain action $a_i(t) \in \mathbb{R}_+$, that is interpreted as the technology level she currently uses. If we were to think of computer technology, then $a_i(t)$ could indicate the speed or capacity of the computer used by i at t. Every agent i is assumed to interact with the individual to the right and to the left of her (i.e. $i + 1$ and $i - 1$, these indices interpreted as "modulo n"). Out of each of these two interactions, she obtains corresponding payoffs, $\psi(a_i(t), a_{i+1}(t))$ and $\psi(a_i(t), a_{i-1}(t))$, with $\psi(\cdot)$ being called the *payoff function*.

In our simulations, we will postulate a stylized payoff function of the following form:

$$\psi(a, a') = \begin{cases} a - k_1 \left(1 - e^{-(a-a')}\right) & \text{if } a \geq a' \\ a - k_2 \left(1 - e^{-(a'-a)}\right) & \text{if } a < a' \end{cases} \tag{1}$$

for some $k_1, k_2 > 0$. By way of illustration, we may think of the interaction taking place between any two agents as consisting of the completion of a certain project (e.g. launching a new product through firm partnership). Then, the crucial aspect of the interaction that (1) models is that there is an advantage to compatibility (i.e. similarity) of technological levels.

More specifically, payoffs (e.g. profits) accrue from the composition of two components. On the one hand, there is the *base payoff* obtained by individual using technology level a, that is assumed increasing in this level – for simplicity, we just make it *equal* to this level, thus in essence parametrizing technological level through the base payoff it induces. On the other hand, we have the costs associated to different degrees of technological incompatibility with the neighbors' technologies. These costs are derived from two alternative sources: (i) the agent is too advanced relative to her neighbors; (ii) the agent is too backwards. In either case, the bilateral interaction in question is assumed partially disrupted, which leads to the waste of some valuable resources. The magnitude of each of these two possible costs is parametrized, respectively, by k_1 and k_2. As it turns out, only the difference $k_1 - k_2$ will be found to play an important role in the analysis.

III. THE DYNAMICS

The process starts from some given initial state $[a_i(0)]_{i=1}^n$. [15] At each $t = 1, 2, \ldots$, a single agent $\iota(t)$ is randomly chosen to update her technological level. (This update may be conceived of as an "innovation".) The agent thus chosen is subject to an upward shift in her technological level that passes from $a_i(t-1)$ to $a_i(t-1) + \tilde{\sigma}$ where $\tilde{\sigma}$ is a i.i.d. random variable, that will be typically postulated uniformly distributed on the interval $[0,1]$. After such random update is performed at t on agent $\iota(t)$, the following adjustment process ensues, with its steps indexed by $q = 0, 1, 2, \ldots$

At $q = 0$, we make $\alpha_j^0(t) = a_j(t-1)$ for $j \neq \iota(t)$ and $\alpha_{\iota(t)}^0(t) = a_{\iota(t)}(t-1) + \sigma_t$, where σ_t is the realization of the random variable $\tilde{\sigma}$ at t. Subsequently, at every $q = 1, 2, \ldots$, some agent i is randomly selected and is given the opportunity to adjust her former technology level $\alpha_i^{q-1}(t)$. Then, she chooses the technology level that solves the problem:

$$\max_{\alpha \geq \alpha_i^{q-1}(t)} \left[\psi(\alpha, \alpha_{i+1}^{q-1}(t)) + \psi(\alpha, \alpha_{i-1}^{q-1}(t))\right]. \tag{2}$$

The solution of this problem is then made $\alpha_i^q(t)$. For all other agents $j \neq i$, $\alpha_j^q(t) = \alpha_j^{q-1}(t)$.

This process continues for all required q until a certain $\bar{q}$ is reached in which it can be ensured that no agent wants to perform any adjustment in her technological level. By the construction of the process and the properties of the payoff function, this $\bar{q}$ is certain to be finite. Then, we make $a_i(t) = \alpha_i^{\bar{q}}(t)$ and the process turns to the next "period" $t+1$, where an analogous chain of initial update and ensuing adjustment is conducted.

At each t, the adjustment process that restores stability after the initial update will be called a (technological) *avalanche*. In our simulations, we shall be interested in quantifying the size $s(t)$ of each avalanche at t as follows:

$$s(t) \equiv \#\{i : a_i(t) \neq a_i(t-1)\}, \tag{3}$$

where $\#\{\cdot\}$ stands for the cardinality of the set in question. We shall also concern ourselves with the total advance triggered by the avalanche, as given by:

$$H(t) \equiv \sum_{i=1}^{n} [a_i(t) - a_i(t-1)]. \tag{4}$$

As explained below, these magnitudes are seen to display interesting regularities in our simulations. Of course, the specific details of these regularities depend on the underlying parameters of the model – specifically, on the incompatibility costs captured by k_1 and k_2.

IV. COMPUTER SIMULATIONS

We have performed computer simulations for the dynamics outlined above on one-dimensional lattices. In order to understand the effect of the different parameters involved, we consider different system sizes, noise levels, k's, and initial and boundary conditions.

We focus on three types of complementary results:

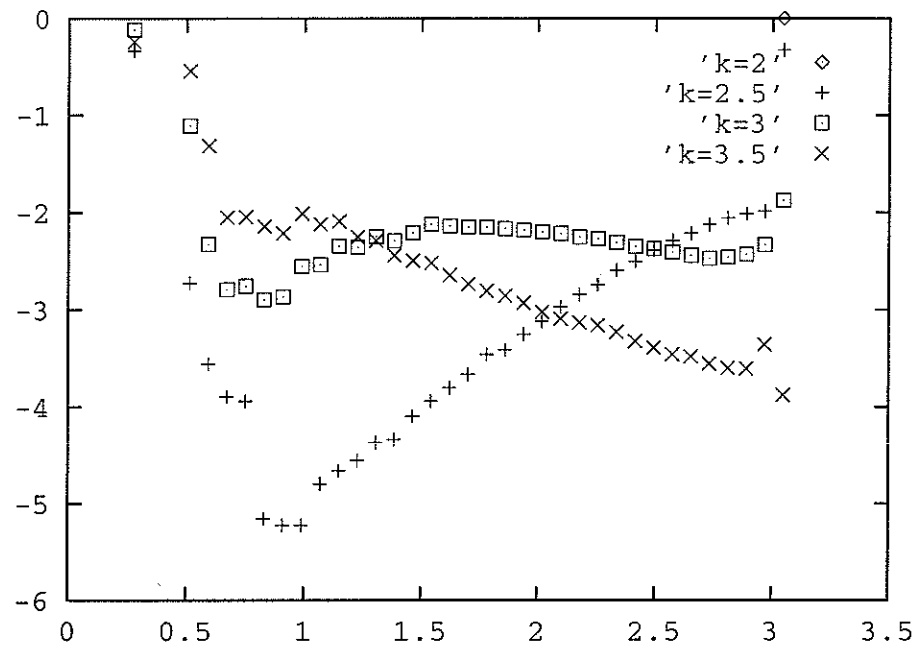


FIG. 2: Logarithm of the probability of having an avalanche of size s vs. logarithm of the size for k=2, 2.5, 3, 3.5. The length of the system is kept fixed at 1024.

- size distributions of technological avalanches
- technological-index profiles
- roughness of these profiles

A Size distributions technological avalanches

Let $k = k_1 - k_2$. As shown below (cf. Section V), k is the essential cost parameter underlying the long-run behavior of the system. We have obtained the (avalanche) size distributions for different values of k and different values of n (the latter is considered in order to check for possible finite-population effects). To get a better statistics we have followed a binnig procedure, i.e. we consider intervals of exponentially increasing length as representatives of size. The outcome of these simulations is summarized in Figs. 2-4.

In Fig. 2 we show the size distribution for "small" values of k, where we can notice the transition from a regime where all the avalanches are system-size wide (we will call this regime supercritical or synchronized) to a regime where the avalanche size obeys a power-law distribution. The latter corresponds to a critical regime in the sense used in statistical physics, i.e. it describes a process in which there are no characteristic length scales. For intermediate values, we notice a transition from distributions that display a positive exponent (and thus keep some trends of the synchronized regime) to distributions with a negative exponent.

Fig. 3 shows size distributions that are clearly in the critical region, the corresponding exponent changing with k. It is worth noting that, as $k \to \infty$, any interaction between neighboring sites vanishes and one obtains a process of so-called random deposition, a very well known process in the study of surface growth (see for instance [9]).

In Fig. 4, it is confirmed that criticality is not a finite-size effect since power laws are preserved (with exponents that depend on k) over three orders of magnitude. However, the distribution tails do display finite-size effects due to the periodic boundary conditions used throughout the paper. In Ref. [10] we have also confirmed that the initial decay is exponential up to a value of $s_0 = 3$ that is independent of the size of the system. In order to do that we have computed the probability that an agent adjusts her technological level after an innovation has been introduced in the system.

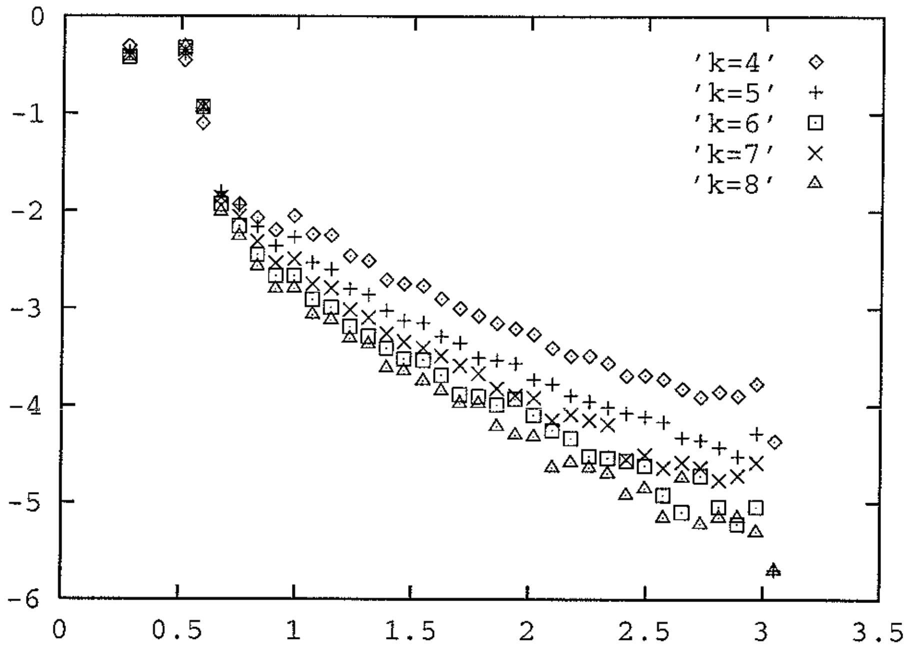


FIG. 3: Same as Fig. 2. for k=4, 5, 6, 7, 8

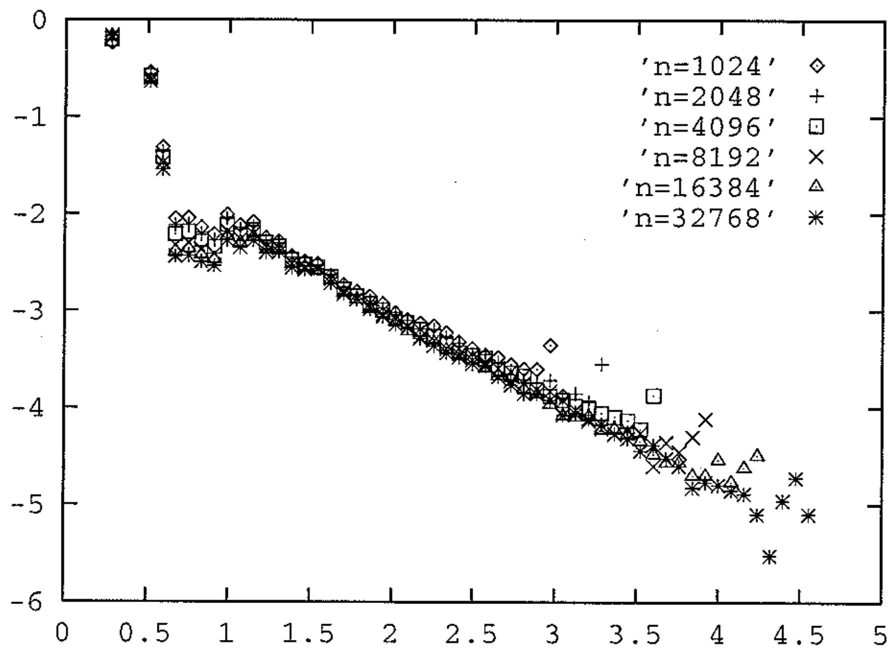


FIG. 4: Same as Fig. 2. for a fixed value of k=3.5 and different system sizes 1024, 2048, 4096, 8192, 16384, and 32768.

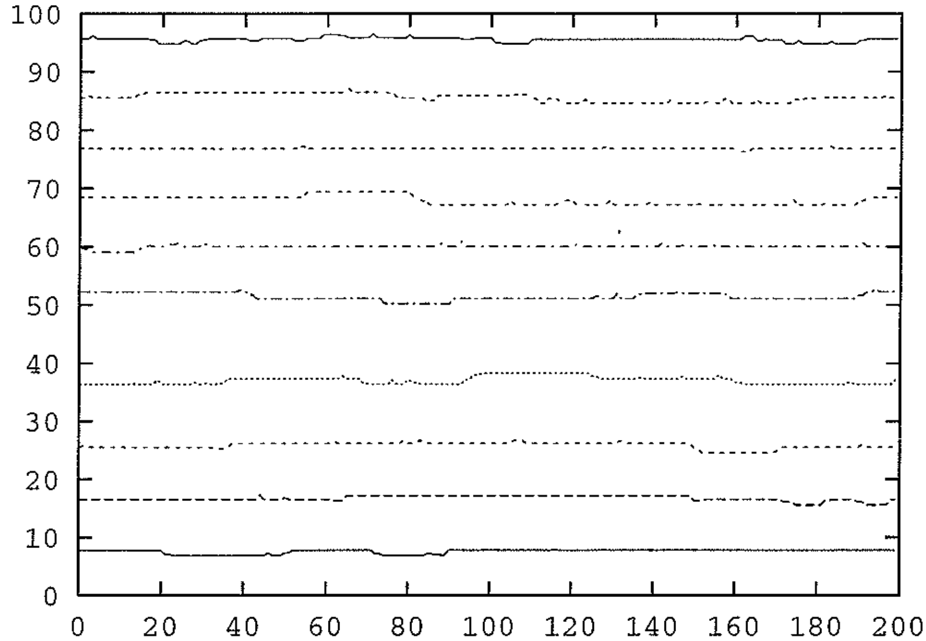


FIG. 5: Technology level profile for k=3.

In sum, we observe that the system undergoes a transition from a synchronized (supercritical) behavior to a non-interacting (subcritical) regime, the system behaving in a critical manner across a wide intermediate range of parameters, no parameter fine-tuning needed to achieve such criticality.

B Technological profiles

Here we focus on the evolution of small lattices (n = 200) along a short span of time (1000 exogenous updates, plotted at intervals of 100), starting from a flat distribution of technology indices. The figures below show again the transition between the different regimes. For $k < 3$, the profiles are essentially flat since, in this case, very large avalanches typically occur after an update. On the opposite case, when k is large, for endogenous adjustments to occur a very large local gradient is needed and hence the interface profile is typically very rough; we will quantify this roughness in the following subsection.

C Profile roughness and technological distribution

In surface growth problems, it is common to quantify the roughness of an interface by its width, which is then related by means of a scaling hypothesis to the surface correlation functions. The width is defined by the root mean square fluctuations in the technology level profile:

$$W(t,n) = \sqrt{\overline{a(t)^2} - \overline{a(t)}^2}, \tag{5}$$

where the overline denotes spatial average, i.e.

$$\overline{a(t)^q} = \frac{1}{n}\sum_{i=1}^{n}(a_i(t))^q. \tag{6}$$

When there are no characteristic time or length scales in the dynamical evolution of a system then this width is expected to have the following behavior

$$W(t,n) = t^\beta \Phi(t/n^z) = n^\alpha \phi(t/n^z) \tag{7}$$

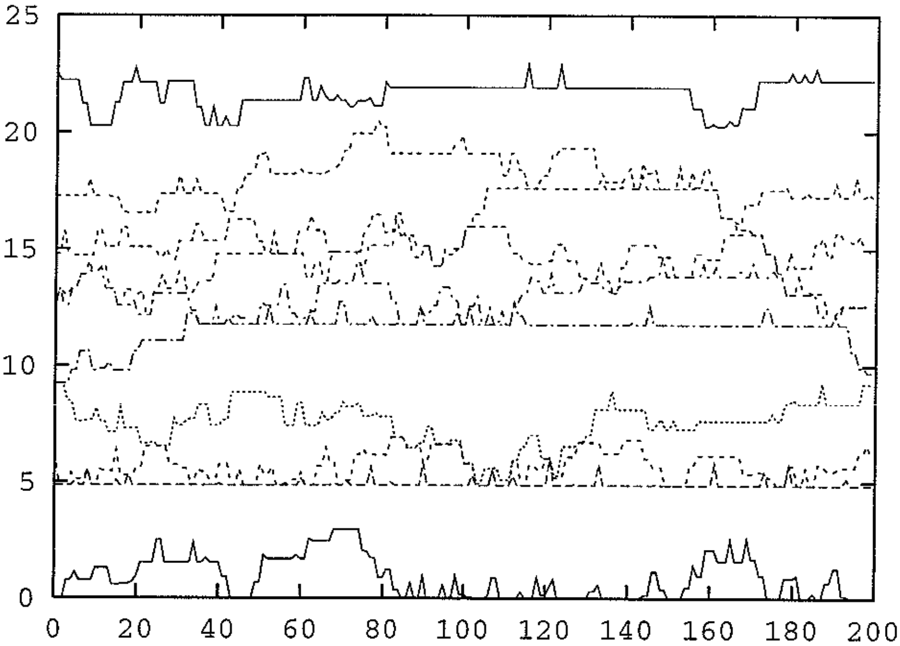


FIG. 6: Same as Fig. 5 for k=3.5.

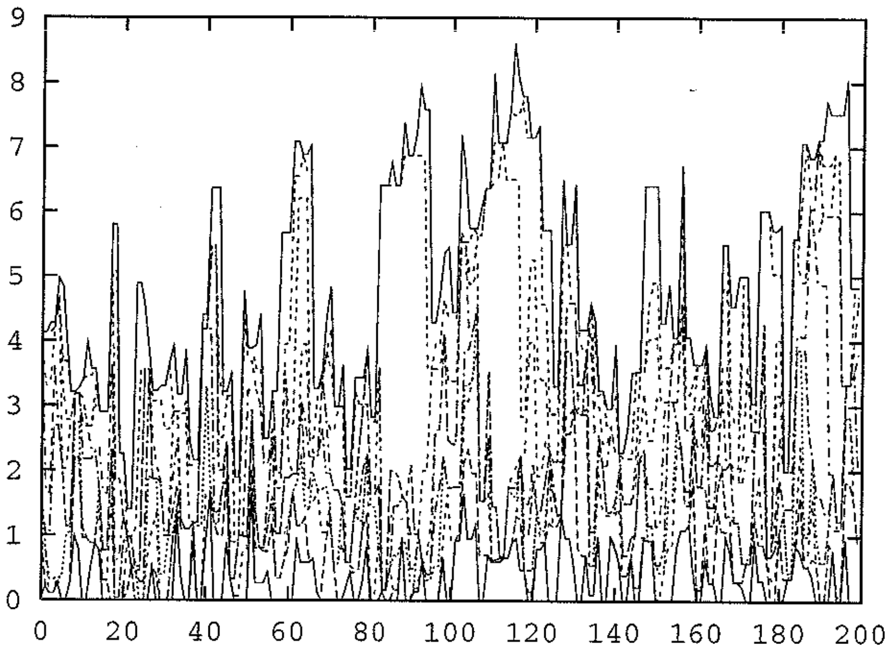


FIG. 7: Same as Fig. 5 for k=8.

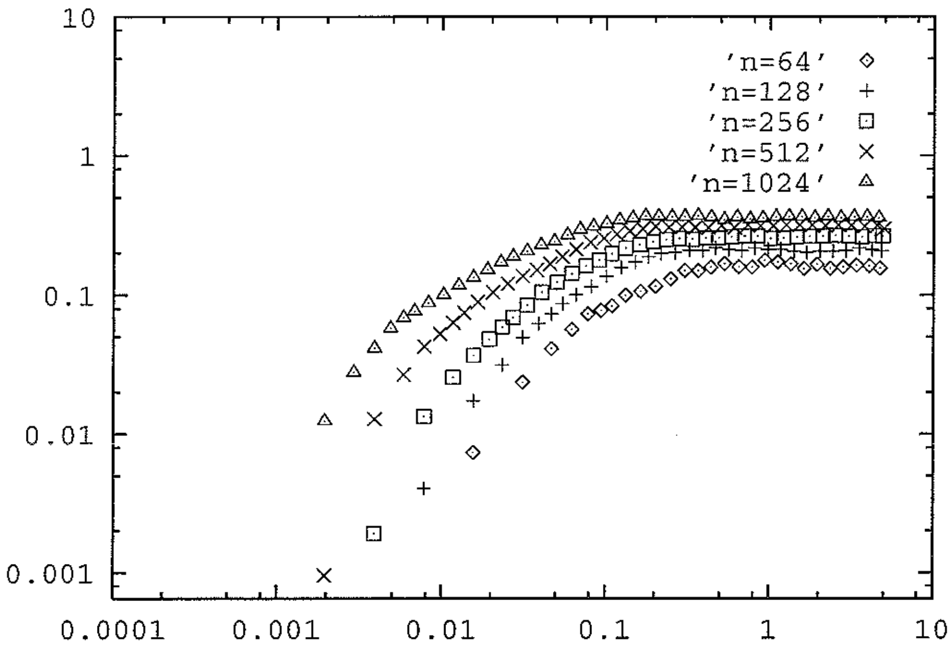


FIG. 8: Squared width of the technology profile as a function of time, for different system sizes and k=3.

where Φ and ϕ are scaling functions, and β, α, and z are the critical exponents describing the critical properties of the system. In particular, z is called the *dynamical exponent* and describes the time approximately needed for the system to reach saturation (i.e. the state where its roughness does not increase any longer); on the other hand, α is called the *roughness exponent* and determines the growth of the interface in terms of the system length; finally, β is usually called the *growth exponent* and characterizes the time-dependent dynamics of the roughening process.

It is interesting to note that the formerly indicated exponents are not independent. Specifically, it is easy to check that simple manipulations lead to the relation:

$$z = \alpha/\beta, \tag{8}$$

which is typically labelled a *scaling law* in statistical mechanics. Recent simulations show that local fluctuations do not scale as global fluctuations [10, 11] giving rise to what is known as *anomalous scaling* [12].

The figures below depict the evolution of the width of the system, as defined in (5). Note that since we are only interested in the *statistical* properties of the profile dynamics, the evolution of this magnitude is averaged over 1000 independent runs.[16]

Figs. 8-10 show that the "technological roughness" of the system evolves with time and increases with system size according to the scaling laws introduced above. One also observes that the technological interface is made rougher as k increases. The time scale is always measured as the number of external upgrades per site.

Overall, the above simulation results suggest that the system has a well-defined level of technological heterogeneity on which the process settles in the long-run, independently of initial conditions. In fact, it is precisely such underlying scope for inter-agent technological differences that underlies the other long-run regularities (in particular those concerning size distributions) that have been depicted in Fig. 2-4. In a sense, such asymptotic roughness is nothing but a different manifestation of the other phenomenon studied before. They all are the outcome of a dynamic process of technological advance that despite of (or rather, because of) microeconomic erraticness produces average long-run regularities in a wide range of different dimensions.

V. THE PARAMETERS OF THE MODEL

Given the payoff formulation adopted in (1), k_1 and k_2 are the only (cost) parameters of the model. As advanced, we want to argue that, in order to understand the rise of critical behavior in the system, only the difference $k \equiv$

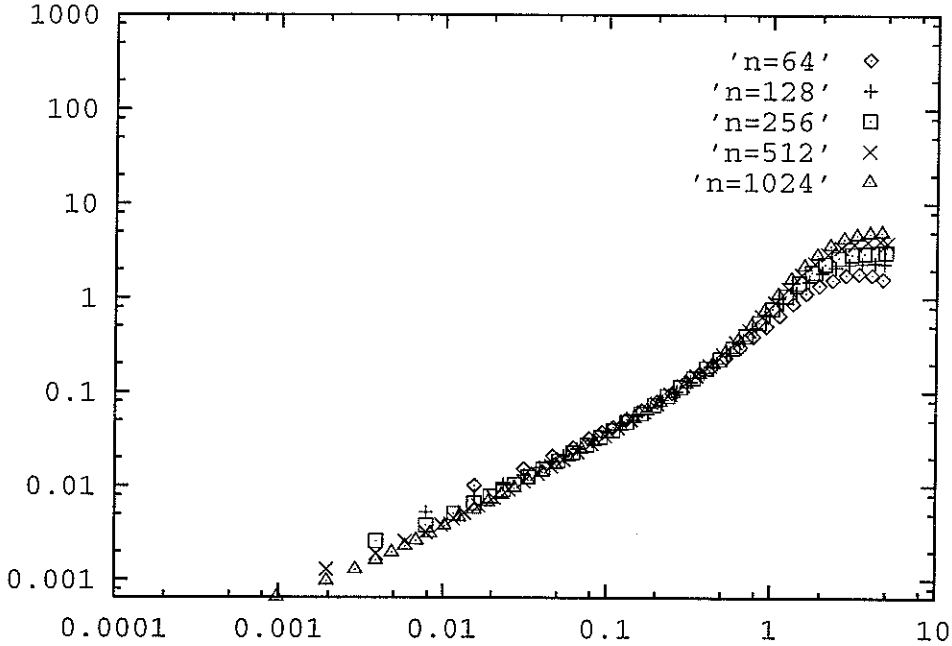


FIG. 9: Same as Fig. 8 for k=6.

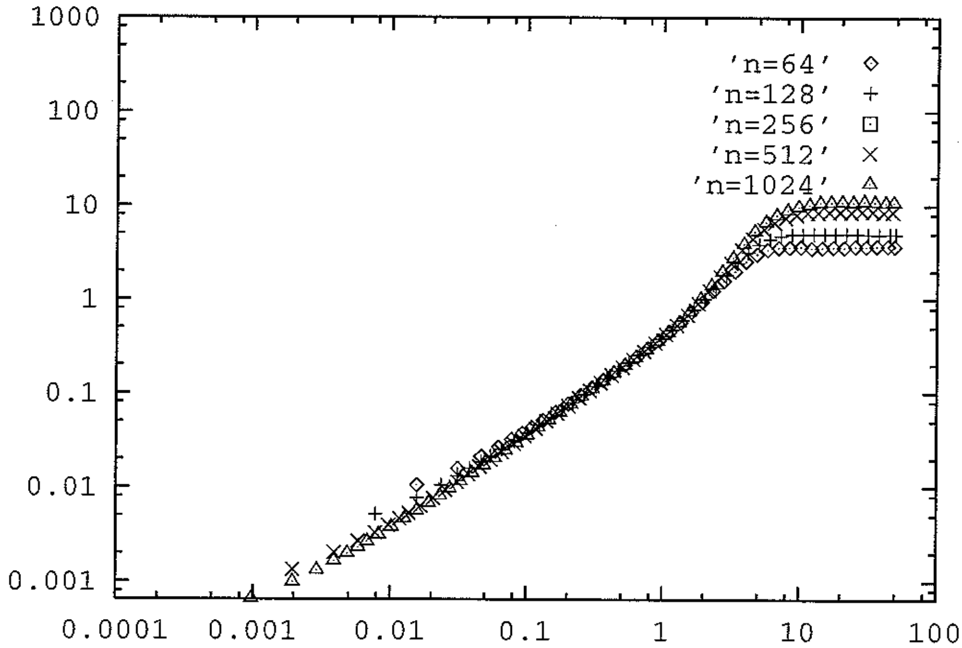


FIG. 10: Same as Fig. 8 for k=9. Notice in this case that we have changed the time scale, since the system needs much more updates to reach saturation.

$k_1 - k_2$ needs to concern us. Intuitively, k reflects the cost difference resulting from "downwards incompatibility" (i.e. being too advanced) as compared to that derived from "upwards incompatibility" (i.e. being too backwards). To see that this cost difference must be the key consideration involved, consider a synchronized state $[a_i(t)]_{i=1}^n$ where every agent displays an essentially identical technological level, i.e. $a_i(t) \approx a_{i+1}(t) \approx \hat{a}$ for all $i = 1, 2, ..., n$. Now suppose that an innovation update of magnitude Δ occurs for some particular agent i_o. When will this update lead the system into a new synchronized state? This requires that the payoff to any agent $i \neq i_o$ of adopting $\hat{a} + \Delta$ when at least one of her neighbors has done so is higher than if she were to remain at level $\hat{a}$. Thus, if we focus on the only non-trivial case where one neighbor adopts $\hat{a}$, the relevant inequality is:

$$2(\hat{a} + \Delta) - k_1(1 - e^{-\Delta}) > 2\hat{a} + \Delta - k_2(1 - e^{-\Delta}),$$

or equivalently:

$$k \equiv k_1 - k_2 < \frac{2\Delta}{1 - e^{-\Delta}}. \tag{9}$$

If (9) holds, the innovation will lead the system into a new synchronized state at the common level $\hat{a} + \Delta$, which indicates that, in the long-run, the process will display supercritical behavior. Critical behavior will start to set in only when the inequality converse to (9) begins to apply. That is, when

$$k > k^* \equiv \frac{2\Delta}{1 - e^{-\Delta}}$$

For example, if all innovation updates were to take place at a fixed magnitude $\Delta = 1$, we would have $k^* = 3.164$ – note how the transition from a supercritical behavior to a critical behavior takes place around this value in Fig. 11. However, for innovation updates that are uniformly distributed on the interval $[0, 1]$, the threshold on k when supercritical behavior must start to loose hold is $k^* = 2$. This latter transition can be observed in Fig. 12.

VI. CRITICALITY AND THE RATE OF TECHNOLOGICAL PROGRESS

We have shown that there is a wide range of parameters (specifically, it is enough in every case that k be somewhat larger than 3) within which critical behavior arises. This raises the question of whether there is some rationale that may justify that this parameter range should indeed be attained.

We now provide one such justification. Suppose k may be chosen endogenously as the value that maximizes the rate of average technological progress (see below for a description of this discretionary choice as a decision of economic policy). Then, if we count as a "period" every time an adjustment takes place (either through exogenous update or internal adjustment), it is clear that the rate ρ of average technological progress may be defined as follows:

$$\rho = \lim_{T \to \infty} \frac{\sum_{t=1}^{T} H(t)}{\sum_{t=1}^{T} s(t)}, \tag{10}$$

where $H(t)$ and $s(t)$ are defined above. Denote by $\rho(k)$ the rate associated to some given value of k. Our simulations show that $\rho(k)$ is maximized within the relatively narrow range $k \in [3, 5]$ for which critical behavior is obtained (compare Figs. 2 and Fig. 12).[17]

In the following section, we shall provide an analytical argument for this conclusion. As explained, it formalizes the heuristic idea that the dynamic performance of the system is optimized at the "edge between chaos and order". In our case, this may be identified with the thin region of the state space where the transition from super-critical to critical behavior takes place. For, as explained in Subsection IV A, it is precisely in this region that the long-run dynamics of the system turn from exhibiting little structure to satisfying clear-cut power laws. Another point that deserves some comments is the fact that ρ varies smoothly with k for a uniformly distributed noise whereas it does it discontinuously for a fixed noise; this is related to the nature of the local gradient that has to be broken.

In [14] we have discussed how the maximum rate of technological progress can be related to the size distributions of avalanches and to the induced distributions of technological advances. First, assuming that both probability distributions obey a power-law:

$$P(s) \sim 1/s^{\gamma} \tag{11a}$$

$$P(H) \sim 1/H^{\beta} \tag{11b}$$

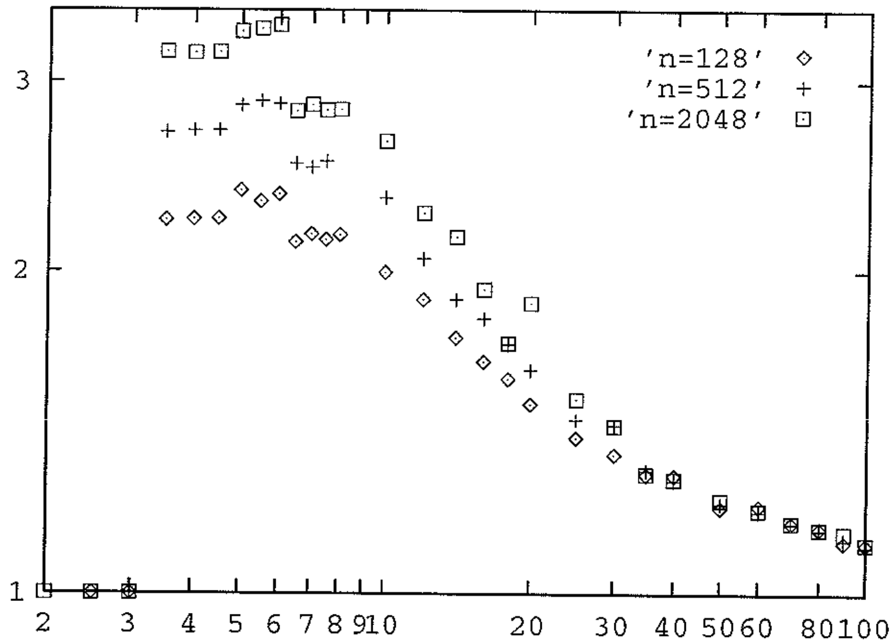


FIG. 11: Rate of technological advance as a function of k, in a log-log scale, for three different values of the length. For each run we have generated 64*L avalanches and averaged over 10 independent realizations of the noise, unless for L=2048 where only two independent realizations have been considered. The exogenous update is fixed to 1.

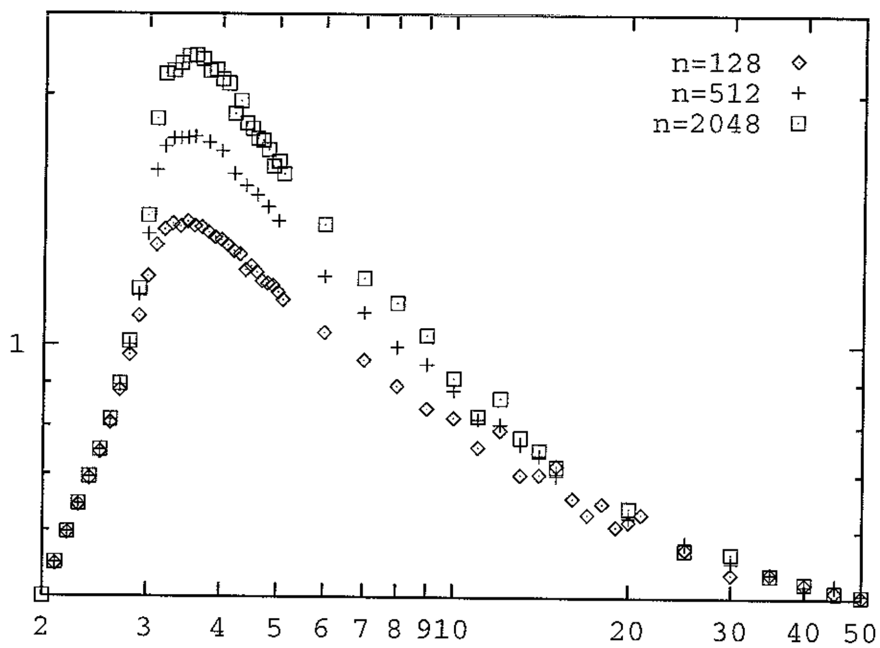


FIG. 12: Same as Fig. 11 but in the present case the exogenous update is chosen from a uniform distribution [0,1].