

# An Internet Local Routing Approach Based on Network Structural Connectivity

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**Abstract**—Internet is one of the largest synthetic complex system ever built. It consists in a collection of more than 30,000 networks each one known as an Autonomous System. In the last few years, Internet is experiencing an explosive growth that is compromising its navigation scalability due to its dependence on the Border Gateway Protocol (BGP). The BGP routing protocol requires to maintain an updated partial view of the network topology, involving a huge amount of data exchange and significant convergence times. The scale-free topology of Internet makes complex network theory the natural framework to analyze its problems and propose solutions. Here, we present a local alternative to BGP based on complex networks. Our approach uses the linear projection of the modular structure of the network to construct a navigable map of the Internet. This map guarantees a high reliability over time on the actual evolving network, in the sense that projection changes are negligible. The simulation results show that we are in high percentage close to optimal paths.

## I. INTRODUCTION

Internet is one of the largest synthetic complex system ever built, comprising a decentralized collection of more than 30,000 computer networks from all around the world. Each one of these networks is known as an Autonomous System, this is, a network or group of networks under the management of a single authority. Two ASs are connected if and only if they establish a business relationship (customer-provider or peer-to-peer relationships), making the Internet a self-organized system.

In the last few years, Internet is experiencing an explosive growth that is compromising its navigation scalability [1] due to its dependence on the Border Gateway Protocol (BGP). BGP is the routing protocol of the Internet and exchanges information of reachability through the network to maintain a vector with end-to-end paths. This vector is called the routing table and it exhibits an exponential growth, i.e. it scales at least linearly with the network size. In addition, its update involves a huge amount of data exchange and significant convergence times of up tens of seconds. With the growth of the network this problem is compounded daily. These poor scaling properties have been studied by the research field of Compact Routing (see [2] for a review). In the presence of topology dynamics, a better scaling on Internet-like topologies is fundamentally impossible: the amount of messages per topology change can not grow slower than linearly.

The scale-free topology of Internet [3] makes the complex network theory the natural framework to analyze its problems

and propose solutions.

From the perspective of artificial intelligence, the search of a solution in a dataset modeled as a network has been a widely studied problem. Having enough information, we can use an informed search method to guide a search process with more or less success. Our initial hypothesis is that the modular structure inherent to real complex networks provides a useful information that can be exploited to design efficient search algorithms. Here, we present a local alternative to the BGP routing protocol based on a linear projection of the modular structure of the network. This projection is used to construct a navigable map of the Internet that guarantees a high reliability over time on the actual evolving network, in the sense that projection changes are negligible.

This paper is organized as follows. Section 2 introduces some concepts of complex network theory necessary to understand this work. In Section 3 we present the methodology to construct a map that reflects the modular structure of a complex network. Next section explains the heuristic routing algorithm we propose to navigate in complex networks. Section 5 shows the simulation results of the routing algorithm in the Internet network, and Section 6 discusses about the reliability of the proposed framework. Then we present the conclusions.

## II. INTRODUCTION TO COMPLEX NETWORKS

A complex network is a graph whose connectivity is not regular. Let assume we have  $N$  nodes and  $L$  links. Two vertices  $v$  and  $u$  are *adjacent*, or neighbors, if they have an edge or link connecting them. If every link has a weight or label associated the network is called *weighted network*, otherwise, will be called *unweighted*. The *degree* of a node is the number of connections a node has to other nodes.

A *path* in a network is a sequence of vertices  $v_1, v_2, \dots, v_n$  such that from each of its vertices there is a link to the next vertex in the sequence. The *length* of the path between  $v_1$  and  $v_n$  is then the sum of the weights of the links in the path, which is  $n - 1$  in unweighted networks.

### A. Scale-free networks

Several empirical results demonstrate that many large networks are scale free, that is, their degree distribution follows a power law in  $k$ . So the probability that a given node in the

network has  $k$  connections is

$$P(k) \sim k^{-\gamma}. \quad (1)$$

In 1999, Barabási and Albert presented some data and formal work that has led to the construction of various scale-free models that, by focusing on the network dynamics, aim to offer a universal theory of network evolution [4]. The important question is then: what is the mechanism responsible for the emergence of scale-free networks? While the goal of the former models is to construct a graph with correct topological features, the modeling of scale-free networks will put the emphasis on capturing the network dynamics. In the first place, random and small-world networks [5], [6] assume that graphs start with a fixed number  $N$  of vertices that are then randomly connected or rewired, without modifying  $N$ . In contrast, most real-world networks describe open systems that grow by the continuous addition of new nodes.

Second, most real networks exhibit *preferential attachment*, the likelihood of connecting to a node depends on the node degree (e.g. a web page will more likely include hyperlinks to popular documents with already high degrees).

### B. Modular structure

The modular structure refers to the clustering of nodes in communities, groups of nodes in the network more connected between them than with the rest of the network. Whatever strategy is applied to detect these groups, it must be *blind* to content, and only *aware* of structure.

A widespread approach to quantify a given configuration into communities was proposed in [7]. This measure is known as *modularity* and it rests on the intuitive idea that random networks do not exhibit community structure:

$$Q = \sum_{\alpha} (e_{\alpha\alpha} - a_{\alpha}^2). \quad (2)$$

where  $e$  is a matrix where the elements  $e_{\alpha\beta}$  represent the fraction of total links starting at a node in community  $\alpha$  and ending at a node in community  $\beta$ , and  $e_{\alpha\alpha}$  is the fraction of links starting and ending in community  $\alpha$ . The vector  $a_{\alpha}$  is the sum of any row of  $e$ , i.e. the fraction of links connected to community  $\alpha$ , and  $a_{\alpha}^2$  is the expected number of intra-community links.

Algorithms which optimize this value yield good community structure compared to a null (random) model. The problem is that the partition space of any graph is huge (the search for the optimal modularity value is a NP-hard problem), and one needs a guide to navigate through this space and find maximum values. Some of the most successful heuristics are outlined in [8], [9], [10], [11]. In [12] a comparison of different methods is performed, see also [13]. Modularity-based methods have also been extended to analyze the community structure at different resolution levels [14].

## III. NETWORK MAPPING

The famous Milgram's small-world experiment revealed that there is something special in the structure of many strongly

clustered networks: without a global view of the network, a message can be routed efficiently between any pair of nodes [15]. Our initial hypothesis is that the community structure, that provides meaningful insights on the structure and function of complex networks, is an important actor in these routing properties. To exploit the modular structure of networks we need to analyze the contribution of each node to the communities. The object of this analysis is defined as the contribution matrix  $C$ , of  $M$  communities to  $N$  nodes.

$$C_{i\alpha} = \sum_{j=1}^N W_{ij} S_{j\alpha} \quad (3)$$

where  $W_{ij}$  is the graph matrix, whose elements  $W_{ij}$  are the weights of the connections from any node  $i$  to any node  $j$ ; and  $S_{j\alpha}$  is the partition matrix, where if node  $j$  belongs to community  $\alpha$  then  $S_{j\alpha} = 1$ , otherwise  $S_{j\alpha} = 0$ .

Unfortunately, the study of this matrix involves a huge amount of data. To reduce this problem we propose to analyze the contribution of each node of a network to communities using the projection technique introduced by Arenas *et al.* [16]. This projection is based on a rank 2 truncated singular value decomposition (TSVD) and constructs a plane  $\mathcal{U}_2$  where each node  $n$  has a coordinate pair or contribution projection vector  $\tilde{v}_n$ . This two-dimensional plane reveals the structure of communities and their boundaries, and we will use it as the navigable coordinate system of the complex network.

For each coordinate pair we calculate the polar coordinates  $(R_n, \theta_n)$  where  $R_n$  is the length of the contribution projection vector  $\tilde{v}_n$ , and  $\theta_n$  is the angle between  $\tilde{v}_n$  and the horizontal axis. To interpret correctly this outcome we need to know also the intramodular projection  $\tilde{e}_{\alpha}$  of each community, the distinguished direction line of the projection of its internal nodes (those that have links exclusively inside the community).

As an example, Figure 1 shows the network of Arpanet in July 1976. We can build the contribution matrix of this network with the communities presented and project this matrix in the plane  $R$ - $\theta$  as shown in Figure 2.

With these values, we can compute a new pair  $(R_n, \phi_n)$ , where

$$\phi_n = |\theta_n - \theta_{\tilde{e}_{\alpha}}|, \quad (4)$$

and the new values

$$R_{\text{int}} = R \sin \phi, \quad (5)$$

and

$$R_{\text{ext}} = R \cos \phi. \quad (6)$$

Here,  $R_{\text{int}}$  informs about the internal contribution of nodes to their corresponding communities, and  $R_{\text{ext}}$  reflects the boundary structure of communities. Both values,  $R_{\text{int}}$  and  $R_{\text{ext}}$  are the basis of our routing framework.

## IV. LOCAL ROUTING

To exemplify our local routing proposal we use a classical example in artificial intelligence aimed to find the shortest path between two cities in Romania, from Arad to Bucharest [17].



Fig. 1. Arpanet network at July 1976. Color of nodes correspond to the communities detected using Extremal Optimization [11] implemented in [13].

Let us assume that we know the straight-line distances between all cities of Romania to Bucharest and a road map. Having this information, we can use an informed search strategy to find efficiently the shortest path to our destination. In the given example a successful strategy is to chose the neighbor city (with connection by road) whose distance to destination is shorter. Representing the problem as a network, an informed search method chooses a node for expansion based on an evaluation function  $h(n)$ . The function  $h(n)$  is called heuristic function and estimates the cost of all nodes  $n$  connected to the current node. The node with the lowest cost is selected, e.g. minimize the straight-line distance in the above case.

With the community structure,  $R_{\text{int}}$  and  $R_{\text{ext}}$  we have an additional knowledge of our problem that we can use to guide an informed search strategy.

In Algorithm 1 we present our heuristic function approach. Let us assume we want to go from node  $i$  to node  $j$ , and let  $N_i$  refer to the neighbors of node  $i$ . Each node  $k \in N_i$  is a candidate in the path. In the routing process, for each node  $k \in N_i$  we have to compute a cost function and select the candidate that minimizes it in each step. This process is repeated until the destination is reached, the current node  $i$  does not find a feasible successor or a time constraint is violated. We do not allow loops.

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**Algorithm 1** Heuristic routing algorithm

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cost[*]  $\leftarrow$   $\infty$ 
if  $\exists k \in \alpha_j$  then
  for  $\forall k \in \alpha_j$  do
    cost[k]  $\leftarrow$  calculate equation 7
  end for
else
  for  $\forall k$  with  $R_{\text{ext}_j} > 0$  do
    cost[k]  $\leftarrow$  calculate equation 8
  end for
end if
return  $k$  with lower cost[k]

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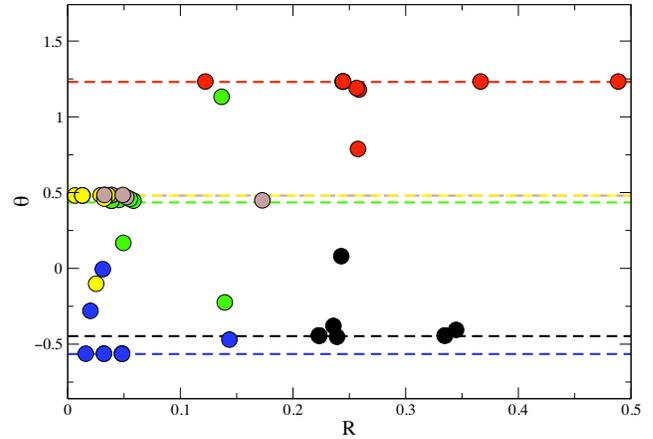


Fig. 2. Projection in the plane  $R$ - $\theta$  of the Arpanet network at July 1976. The dotted lines are the singular directions of each community.

The heuristic algorithm sets two scenarios, when our neighbor  $k$  belongs to the same community  $\alpha_j$  than the destination node  $j$ , and when it doesn't. In the first case, when  $k \in \alpha_j$ , we are interested in finding nodes with an important weight in the community. In the other case, if  $k \notin \alpha_j$ , we seek for nodes near to the boundaries of other communities.

With  $R_{\text{int}}$  and  $R_{\text{ext}}$  we can define a simple cost function. If  $k \in \alpha_j$ , we can define the cost as

$$\frac{1}{R_{\text{int}_k}}. \quad (7)$$

In the other case, if  $k \notin \alpha_j$ , we seek for nodes with high connectivity to other communities as

$$\frac{1}{R_{\text{ext}_k}}. \quad (8)$$

## V. INTERNET ROUTING

To test our local routing framework, we have built a sample network using the AS relationships inferred by CAIDA [18]. These datasets collect the RouteViews BGP table to construct a weekly graph of relationships. Given that the AS relationships were obtained empirically, is sure that this topological view contains errors. In this study we have not considered relationship roles and we have built an undirected network as a first approximation. Our network is also unweighted.

To apply the proposed mapping, first we need to detect the communities of the network. To conduct this, we use mainly the Extremal Optimization [11] community detection algorithm. We apply it recursively decomposing big communities into subcommunities of a desired size, resulting in 349 communities. Then we construct the  $C_{i\alpha}$  matrix and project it on a plane using the TSVD. We have simulated  $10^6$  paths randomly selected. Fig. 3 presents the distribution of path length of our algorithm. For the simple approach presented we have achieved a success rate of 94% and an average path of 7 steps. Given that we are using only local information, the percentage of success is very good. We attribute the

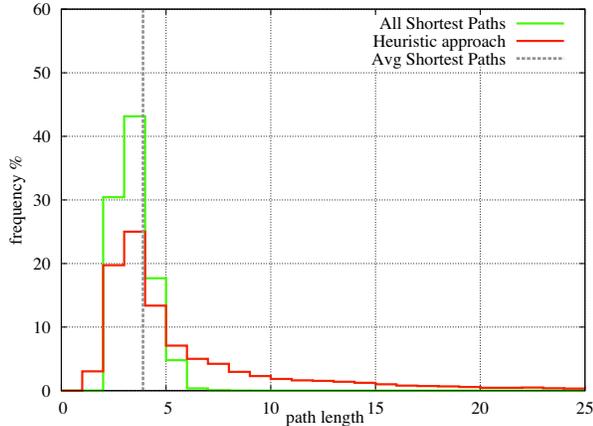


Fig. 3. Distribution of path length of the simulation of  $10^6$  paths.

unreachable destinations to nodes that are very far from any hub of the network.

The average path length performed is bigger than the average of the shortest paths (3.86). This is due to the long tail of our distribution. Nodes with longer paths are, in most cases, internal nodes connected only with other nodes of small degree. Our projection reflects the boundaries of communities of the network but only provides information about the number of connections a node has in the internal topology of its community. This makes that the search problem has difficulties to find the pathway to poorly connected nodes. Consideration of the customer-provider roles will solve partially this problem thanks to the directionality implied in the paths.

In Fig. 4 we analyze the behavior of the algorithm. We have selected the paths of length 3, 4, and 5, that represent more than the 60% of successful paths. Our algorithm first looks for big hubs up to access the destination community, and within it chooses the nodes with largest internal contribution.

## VI. RELIABILITY OF THE ROUTING FRAMEWORK

This study is concerned about the scalability of the Internet routing protocol. In situations where the data is continuously changing, like in an evolving network, a TSVD projection might become obsolete. It is an interesting question whether the TSVD projection of an initial data set is reliable. In our earlier work [19] we defined two measures to quantify the differences between a sequence of computed TSVD projections of growing Barabási-Albert’s scale-free networks. In that situation, we proved that the stability of a TSDV map is very high when considering neighborhood stability (note that  $R_{\text{int}}$  and  $R_{\text{ext}}$  are relative modules). Thanks to this, under topology dynamics, addition and removal of nodes in the network can be done without recalculating the projection. For more details on this we refer the reader to [16].

Here, we experimentally show that such stability is also reflected in our routing framework performance. We have chosen two snapshots of the ASs network, of June and December 2009, to construct and project two undirected and unweighted networks. Our routing algorithm achieves similar

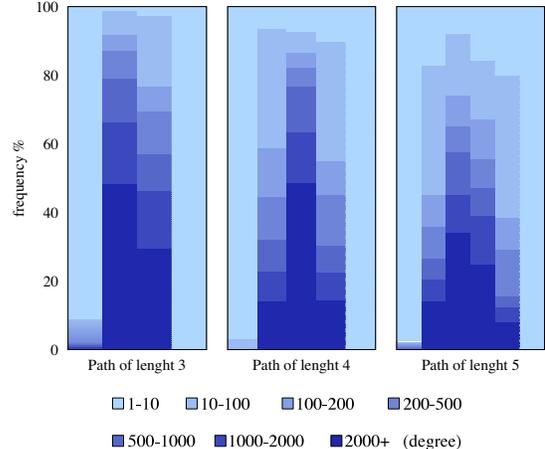


Fig. 4. Distribution of degree nodes in each step of path for  $10^6$  simulations. In  $x$  axis each column corresponds to a node in the path.

success rates in both networks with comparable path length distributions, as shown in Fig. 5. This reinforces the idea that our schema is very robust against evolving data.

## VII. CONCLUSIONS AND DISCUSSION

In an unstructured network an algorithm based on hubs’ transit will result in a decent routing even improving classical routing techniques [20]. However the use of the modular structure inherent to real complex networks provides a useful information whose exploitation in terms of the projection is competitive with global shortest path strategies. Here, we have presented a heuristic approach that uses the projection of the matrix of contributions of each node to communities to guide the routing process of Internet.

With our proposal we address one of the major problems that compromises the scalability of Internet: its dependence on the Border Gateway Protocol (BGP). Our local alternative to this routing protocol achieves a success rate up to 94% with an average path of 7 steps. Our results guarantees a high reliability over time, in the sense that projection changes are negligible and, therefore, the routing process remains valid.

Recently, Boguñá *et al.* [21] have presented a mechanism that tries to explain the navigability of real networks based on the concept of similarity between nodes. They use statistical inference techniques to assign a pair of coordinates to each node in an hyperbolic metric space. In a Poincare representation, the nodes with high degree go to the center of the space and their angular position is determined by their probability to be connected, thus on a equivalent way to the community structure. At the end, their method uses the same principles as ours: going through the geodesics of their hyperbolic space from any origin node to any destination node is equivalent to finding the hubs of the network and going closer to the destination community.

Another recent work that studies the routing process in scale-free networks have been introduced by Lattanzi *et al.* [22]. This study is focused on social networks and uses the

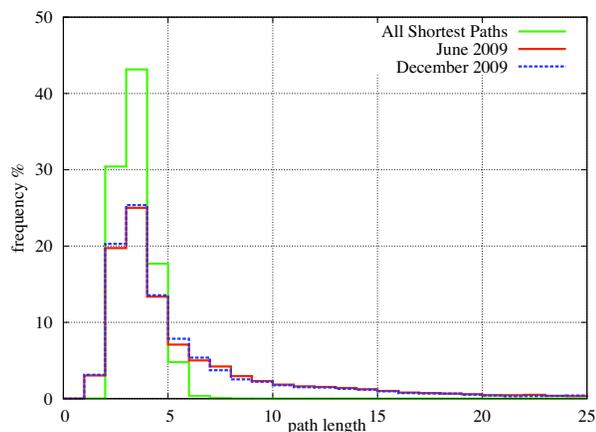


Fig. 5. Comparison of path length performance of two snapshots of ASs network.

model of *Affiliation Networks* that considers the existence of an *interest space* lying underneath. The search is conducted greedily in this space and their results reinforce our assumption that low degree nodes not connected directly to hubs are hard to find, and that large hubs are essential for an efficient routing process. Contrary to our model, it uses extra information beyond the network topology making it more dependent on the dataset.

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