Using aggregation operators to personalize agent-based medical services

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Abstract. In previous papers we introduced HeCaSe2, a multi-agent system that helps doctors to follow the automatic application of clinical guidelines to patients. In this paper we show how aggregation operators, based on fuzzy logic, may be integrated in this system in order to personalize some of its tasks. These operators take into account the patient preferences when several medical services propose different conditions under which a specific medical test can be performed. The paper describes how different proposals can be rated and ranked, and discusses the influence of two parameters (the set of linguistic preference values and the rating policy) on the results of the aggregation procedure.

1 Introduction

Any computer system designed to work in a medical setting has to take into account different issues; in [1] it was argued that the following ones suggest the appropriateness of the use of *agent technology* in the health care area:

- Heterogeneous data: medical centres generate data from very different sources (e.g. an X-ray image, a blood test, the result of a medical visit, etc.) and it is necessary to integrate them smoothly (e.g. new data should be added easily into the patient's electronic medical record).
- Autonomy: services, departments, medical practitioners and patients are autonomous entities with their own knowledge, beliefs and goals. Any model of the activities within a medical centre should allow these entities to keep their autonomous behavior.
- *Distributed data*: the data related to a patient is usually distributed among different units (services, departments) of a hospital.
- Complex coordination: a medical centre has a large number of (human and physical) resources that have to be managed during careflow (i.e. the workflow processes involved in the provision of care, [2, 3]). All of them play a specific role within the medical centre organisation, and they must coordinate their activities to provide the best possible care to patients.

A clinical guideline (CG) indicates the protocol to be followed when a patient is diagnosed a certain illness (e.g. which medical tests have to be performed on the patient to get further data, or what steps have to be taken according to the results of the tests). Therefore, they provide very detailed information about the resources needed in the treatment of the patient [4]. Its adoption could improve the quality of patient assistance. Unfortunately, guidelines are not used extensively by practitioners due to two main reasons: a) the difficulty to adapt standard CGs to the particularizations of each sanitary centre, and b) the little tuning between the CG and the workflow of the professionals [5]. It can be argued that an agent-based automation of CGs could bring interesting benefits to the health care area, such as the following:

- Doctors are automatically reminded about the steps that should be followed in the treatment of a certain disease, and that reduces the possibility of them making errors or forgetting tasks to be done.
- Agents representing patients, doctors, departments and hospital services can automatically coordinate their activities to provide a fast care (*e.g.* by scheduling different tests to be performed on the patient on his behalf).
- These agents can apply AI techniques to solve their tasks, adding an intelligent component that improves their performance during negotiation and coordination processes.

The aim of the paper is to show how we can integrate aggregation techniques, based on fuzzy logic, into an agent-based system that provides health care services. A Multicriteria Decision Making process has been implemented in order to rank a set of alternatives (*e.g.* different possible dates in which a medical test can be performed) depending on the user's preferences [6].

The rest of the paper is organised as follows. The next section describes an agent-based system $(HeCaSe2)^1$, in which a set of agents has been designed to help doctors to follow the application of guidelines to particular patients. Section 3 explains how agents can use aggregation procedures to analyse a set of appointment proposals from the patient's point of view in order to personalize the system's performance. Section 4 shows the application of the aggregation method to some example data, and it analyzes how the change of some parameters influences in the final results. Finally, the last section details some lines of future work.

2 Guideline-based distributed healthcare system

In previous works ([7]) we presented the main ideas underlying HeCaSe2, an agent-based distributed system that has the aim of easing the application of computer interpretable guidelines on particular patients. HeCaSe2 is a multi-agent system that maps different entities in a healthcare organization (medical centres, departments, services, doctors, patients) as agents with different roles

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and goals. This system provides interesting services both to patients (*e.g.* booking a visit with a doctor, or looking up the medical record) and to doctors (*e.g.* support in the application of a CG to a patient).

Guidelines are used to provide a high level supervision of the activities to be carried out to address a specific pathology. We use PRO*forma* as the language to represent and share guidelines [8]. It defines four types of tasks: *i*) actions, that are procedures that have to be executed outside the computer, *ii*) decisions, that are used to choose a candidate from a given set of options using arguments pro and con, *iii*) inquiries, that are requests for information needed to execute a certain procedure, and *iv*) plans, that are a sequence of sub-tasks taking into account logical or temporal constraints. Thus, a guideline can be defined as a set of plans that are composed by actions, decisions and inquiries.

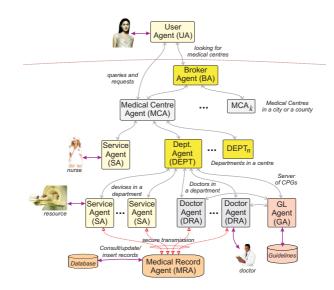


Fig. 1. HeCaSe2 agent-based architecture

The agents in the system (see Fig. 1) coordinate their activities in order to apply a guideline to a patient (always under the supervision of a doctor). The basic steps are the following (more details are given in [7]):

- When the doctor diagnoses that the patient has a certain disease, its associated *Doctor Agent* (DRA) requests from the *Guideline Agent* (GA) the guideline associated to that condition, and it starts to execute it.
- If the guideline needs some medical data, the DRA can request it from the *Medical Record Agent* (MRA), that has access to the electronic medical record of the patient.
- Sometimes an action has to be performed on the patient, or the guideline needs data that is not included in the medical record (e.g. the level of glucose

in blood, which can be known with a blood analysis). In these cases, the DRA has to contact *Service Agents* (SAs), from the same medical centre or other medical centres, that can provide a certain action or clinical test. As there will be different options for each action, we propose in the next section a personalization mechanism, based on aggregation procedures, that receives the service proposals from different SAs and analyses them, taking into account the user's preferences on several criteria. The user receives a ranked list of alternatives, from which he can choose the one that he prefers.

- Once a test has been performed, the result can be sent directly from the SA to the MRA, to be included in the patient's medical record, and the DRA that requested that data is informed so it can follow the application of the guideline on the patient with the new available data.

3 A patient-centered ranking of appointments

In order to build patient-oriented medical systems, we propose the use of intelligent decision-making techniques to help the patient as [9]. In particular, when the doctor, following a guideline, decides that some test must be performed, usually it is the patient who has to find an appointment with an external medical unit that can perform this test, and this is a problem that requires a lot of time and effort from the patient. We propose to automatize this process using the facilities of multi-agent technology and multicriteria decision analysis. In HeCaSe2, when some test t must be performed in one patient, the agent DRA_d begins a call for proposals with the SAs that can do task t. That message is sent to different agents in the medical centre and to SAs in other centres by means of the broker agent (Fig. 1). All SAs that can perform task t seek in its own agenda and send k proposals of possible appointments to the initiator agent DRA_d . A proposal contains the day, location and hour. After receiving all the answers from the SAs, DRA_d completes the proposal with some additional information, building a tuple $p_i = \langle day_of_week, centre, period_day, distance, delay_days \rangle$. In this tuple, we have three linguistic variables: day_of_week, centre (destination medical centre) and *period_day* (morning, afternoon, night), and two numerical ones: distance (kilometres from the origin centre to the destination) and delay_days (days to wait before performing t). Once we have a list of proposals, the system ranks them and only the best n options are shown to the patient. Then, the user can select the most appropriate appointment. The ranking is based on the user's preferences, which are stored in his profile. In the following sections we will describe the user's profile and the ranking technique, which is based on the Linguistic Ordered Weighted Averaging (LOWA) operator [6].

3.1 User's Profile

The user's profile is stored in each User Agent (UA). That profile contains the user's preference information for each attribute. This preference information is given by a utility function, such that $profile_i = \{\mathcal{U}_{atr_h}, h = 1..m\}$. In the profile

we can have linguistic and numerical preferences. Linguistic values are given to categorical attributes, and numerical scores to numerical attributes.

 $\begin{array}{c|c} P = Perfect & (0.925, 0.95, 1.0, 1.0) \\ VH = Very.High & (0.80, 825, 0.925, 0.95) \\ H = High & (0.675, 0.7, 0.8, 0.825) \\ AH = Almost_High & (0.55, 0.575, 0.675, 0.7) \\ M = Medium & (0.425, 0.45, 0.55, 0.575) \\ AM = Almost_Medium & (0.3, 0.325, 0.425, 0.45) \\ L = Low(0.175, 0.2, 0.3, 0.325) \\ VL = Very.Low(0.05, 0.075, 0.175, 0.2) \\ N = None & (0.0, 0.0, 0.5, 0.075) \end{array}$

Fig. 2. A uniformly distributed ordered set of nine labels with its semantics

We will denote $S = \{s_i\}$, and $i \in \{0, \ldots, T\}$ a finite ordered set of T linguistic labels whose semantics is given by fuzzy sets. Each label s_i is defined by a 4-tuple (x_0, x_1, x_2, x_3) , where x_1 and x_2 indicate the interval in which the membership function value is 1, and x_0 and x_3 are the bounds of the definition of a trapezoidal fuzzy membership function. For example, Fig. 2 shows an example considering nine symmetrically distributed fuzzy linguistic labels. Then, in the user's profile we have a utility function $\mathcal{U}_{atr_i}^L$ that associates each possible value of the categorical attribute atr_i to a label in S, indicating its preference score. For numerical attributes, we have a utility function $\mathcal{U}_{atr_i}^N$ that receives the numerical value r of the corresponding attribute, and compares r with the preferred value of the user r_{user} . The utility function of the i^{th} attribute takes $k_i \approx \frac{10}{(max_{atr_i} - min_{atr_i})}$.

$$\mathcal{U}_{atr_i}^N : \mathbb{R} \to [0,1] \qquad \qquad \mathcal{U}_{atr_j}^L : String \to S$$
$$r \to 1/e^{k|r_{user}-r|} \qquad \qquad str \to s_i$$

3.2 The aggregation operator

To rank the set of alternatives we use a decision-making process with two stages: rating and ranking. The rating of each alternative is done in three steps (Fig. 3):

Step 1) All the values describing an alternative, p_i , are transformed into preference values in the domain $([0,1] \cup S)$ by applying the appropriate utility functions \mathcal{U}_{atr_h} as defined in §3.1.

Step 2) The numerical preferences in [0, 1] are transformed into the linguistic domain S by means of a particular numerical-linguistic transformation function defined in [10] (linguistic preferences are left without changes), obtaining the transformed vector called $alt_i = \{a_k\}$ ($a_k \in S$).

Step 3) The linguistic preferences in alt_i are aggregated using the LOWA operator, obtaining a linguistic rating.

Finally, all alternatives can be ranked using the rating values. Then, a filtering is performed to show to the user only the best alternatives, so that he can confirm

one of them (the communication from the UA to the selected SA through the DRA is shown in Fig. 3).

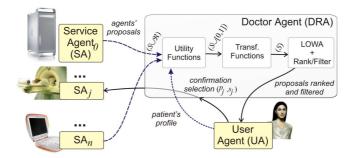


Fig. 3. Aggregation of information process

The problem of aggregating information has been widely studied [11]. There exist several methods to aggregate numerical values as well as linguistic terms. The family of OWA operators are in the class of mean operators, they are idempotent, monotonic and commutative. They are useful to adjust the degree of conjunction and disjunction implicit in any aggregation. This is done by means the use of linguistic quantifiers (expressed as a set of weights) that permits to define different aggregation policies. In [12] different fuzzy majority-based policies are identified, such as "most", "at least half" or "as many as possible".

The LOWA aggregation operator ϕ was defined in [6]. It is an extension of the OWA operator to deal with linguistic variables. The operator ϕ aggregates a set of labels $A = \{a_1, \ldots, a_m\}$, where $a_i \in S$, with respect to a set of weights $W = \{w_1, \ldots, w_m\}$ such that $w_i \in [0, 1]$ and $\sum_i w_i = 1$. Those weights specify the decision-maker policy.

$$\phi(a_1,\ldots,a_m) = W \cdot B^T = \mathcal{C}^m\{w_k, b_k, k = 1,\ldots,m\}$$
$$= w_1 \odot b_1 \oplus (1-w_1) \odot \mathcal{C}^{m-1}\{\beta_h, b_h, h = 2,\ldots,m\}$$

where $\beta_h = w_h / \sum_{2}^{m} w_h, h = \{2, \ldots, m\}$ and $B = \{b_1, \ldots, b_m\}$ is a permutation of the elements of A, such that $B = \sigma(A) = \{a_{\sigma(1)}, \ldots, a_{\sigma(m)}\}$, where $a_{\sigma(j)} \leq a_{\sigma(i)} \forall i \leq j$. C^m is the convex combination operator of m labels; if m = 2, then $C^2\{w_i, b_i, i = 1, 2\} = w_1 \odot s_j \oplus (1 - w_1) \odot s_i = s_k, s_i, s_j \in S, (i \leq j)$ such that, $k = min\{T, i + round(w_i \cdot (j - i))\}$. If $w_j = 1$ and $w_i = 0$ with $i \neq j$, then $C^m\{w_i, b_i, i = 1, m\} = b_j$

4 Analysis of the parameters and results

In this section we give some insight in some points of our personalization method for this particular application. First of all, we will consider the initialization of the utility functions used in the patient's profile $\mathcal{U}_{atr} = (\mathcal{U}_{atr}^N \bigcup \mathcal{U}_{atr}^L)$. When a profile is created, the system does not know any preference, so preferred time and days is considered to be zero and a *medium* preference score is given to each of the qualitative terms. After modifying the patient's profile, we could have a situation like this:

 $\begin{array}{l} delay_days=\langle 0.0\rangle; \ distance=\langle 0.0\rangle; \ centre=\langle (MCBona,N)(MCBorges,M)\\ (MCConst,VH) \ (MCMorell,P) \ (MCGimb,M) \ (MCHospi,AM) \ (MCJaume,L)\\ (MCLlib,L)\rangle; \ day_of_week=\langle (Sun,VL) \ (Mon,L) \ (Tue,M) \ (Wed,VH) \ (Thu,VH) \ (Fri,H)\\ (Sat,VL) \ \rangle; \ period_day=\langle (Morning,H) \ (Afternoon,L) \ (Night,VL) \rangle \end{array}$

The second aspect to analyse is the selection of the set of labels S. We tested the system with different number of labels and with different membership functions (see Table 1). The last point to consider is the LOWA weighting vector W. In Table 1 different configurations of that vector can be observed. Different policies giving priority to the low or to the high values give different results. For instance, alternatives 1 and 3 show the influence of W obtaining both different ratings and positions. Now, we are going to explain the results obtained in the following scenario. Let's consider the previous patient profile and the following 5 possible appointments:

 $\begin{array}{ll} p_0:(2.0,1.2,MCBorges,Wed,Morning) & p_3:(5.0,1.2,MCBorges,Sat,Nigth) \\ p_1:(1.0,8.0,MCConst,Mon,Afternoon) & p_4:(9.0,17.0,MCHospi,Fri,Afternoon) \\ p_2:(4.0,9.0,MCBona,Thu,Afternoon) & \end{array}$

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Entry	Alternatives (A)	Conditions	$Rating(\phi(alt_i))$	Ranking
1	$\begin{array}{l} alt_0 = \langle \mathrm{AH} \ \mathrm{H} \ \mathrm{M} \ \mathrm{VH} \ \mathrm{H} \rangle \\ alt_1 = \langle \mathrm{H} \ \mathrm{VL} \ \mathrm{VH} \ \mathrm{L} \ \mathrm{L} \rangle \\ alt_2 = \langle \mathrm{AM} \ \mathrm{VL} \ \mathrm{N} \ \mathrm{VH} \ \mathrm{L} \rangle \\ alt_3 = \langle \mathrm{L} \ \mathrm{H} \ \mathrm{M} \ \mathrm{AM} \ \mathrm{VL} \rangle \\ alt_4 = \langle \mathrm{VL} \ \mathrm{N} \ \mathrm{AM} \ \mathrm{H} \ \mathrm{L} \rangle \end{array}$		$ \begin{array}{l} \langle \mathbf{H} \rangle \\ \langle \mathbf{AM} \rangle \\ \langle \mathbf{L} \rangle \\ \langle \mathbf{AM} \rangle \\ \langle \mathbf{L} \rangle \end{array} $	$\begin{array}{l} \text{H: } alt_0\\ \text{AM: } alt_1, alt_3\\ \text{L: } alt_2, alt_4 \end{array}$
2	$\begin{array}{l} alt_0 = \langle \mathrm{AH} \ \mathrm{H} \ \mathrm{M} \ \mathrm{VH} \ \mathrm{H} \rangle \\ alt_1 = \langle \mathrm{H} \ \mathrm{VL} \ \mathrm{VH} \ \mathrm{L} \ \mathrm{L} \rangle \\ alt_2 = \langle \mathrm{AM} \ \mathrm{VL} \ \mathrm{N} \ \mathrm{VH} \ \mathrm{L} \rangle \\ alt_3 = \langle \mathrm{L} \ \mathrm{H} \ \mathrm{M} \ \mathrm{AM} \ \mathrm{VL} \rangle \\ alt_4 = \langle \mathrm{VL} \ \mathrm{N} \ \mathrm{AM} \ \mathrm{H} \ \mathrm{L} \rangle \end{array}$	W, at least half $W_5 = (.4, .4, .2, .0, .0)$ S_9 , symmetric $\frac{N_1 \cdot N_1 + c}{2} + \frac{N_1 + N_2 + c}{2} + \frac{N_1 + c}{2} $	$ \begin{array}{c} \langle \mathbf{H} \rangle \\ \langle \mathbf{H} \rangle \\ \langle \mathbf{A} \mathbf{H} \rangle \\ \langle \mathbf{A} \mathbf{H} \rangle \\ \langle \mathbf{A} \mathbf{H} \rangle \\ \langle \mathbf{M} \rangle \end{array} $	H: alt_0, alt_1 AH: alt_2, alt_3 M: alt_4
3	$\begin{array}{l} alt_0 = \langle \mathrm{AH} \ \mathrm{H} \ \mathrm{M} \ \mathrm{VH} \ \mathrm{H} \rangle \\ alt_1 = \langle \mathrm{H} \ \mathrm{VL} \ \mathrm{VH} \ \mathrm{L} \ \mathrm{L} \rangle \\ alt_2 = \langle \mathrm{AM} \ \mathrm{VL} \ \mathrm{N} \ \mathrm{VH} \ \mathrm{L} \rangle \\ alt_3 = \langle \mathrm{L} \ \mathrm{H} \ \mathrm{M} \ \mathrm{AM} \ \mathrm{VL} \rangle \\ alt_4 = \langle \mathrm{VL} \ \mathrm{N} \ \mathrm{AM} \ \mathrm{H} \ \mathrm{L} \rangle \end{array}$	$ \begin{array}{l} W, \ mean \\ W_5 = (.2, .2, .2, .2, .2) \\ S_9, \ symmetric \\ w_1 \cdot w_1 + u_1 + m_1 + m_$	$\begin{array}{c} \langle H \rangle \\ \langle M \rangle \\ \langle AM \rangle \\ \langle M \rangle \\ \langle AM \rangle \\ \langle AM \rangle \end{array}$	$\begin{array}{l} \text{H: } alt_0 \\ \text{M: } alt_1, alt_3 \\ \text{AM: } alt_2, alt_4 \end{array}$
4	$\begin{array}{l} alt_0 = \langle \mathbf{H} \ \mathbf{V} \mathbf{H} \ \mathbf{H} \ \mathbf{V} \mathbf{H} \ \mathbf{P} \rangle \\ alt_1 = \langle \mathbf{V} \mathbf{H} \ \mathbf{V} \mathbf{L} \ \mathbf{V} \mathbf{H} \ \mathbf{L} \ \mathbf{L} \rangle \\ alt_2 = \langle \mathbf{L} \ \mathbf{V} \mathbf{L} \ \mathbf{N} \ \mathbf{H} \ \mathbf{L} \rangle \\ alt_3 = \langle \mathbf{U} \mathbf{H} \ \mathbf{H} \ \mathbf{M} \ \mathbf{V} \mathbf{L} \rangle \\ alt_4 = \langle \mathbf{V} \mathbf{L} \ \mathbf{N} \ \mathbf{M} \ \mathbf{H} \ \mathbf{L} \rangle \end{array}$	W, as many as possible $W_5 = (.0, .0, .2, .4, .4)$ S_7 , symmetric $H^{\text{H}} + H^{\text{H}} + H^{\text{H}} + H^{\text{H}} + F^{\text{H}}$	$\begin{array}{l} \langle \mathrm{VH} \rangle \\ \langle \mathrm{H} \rangle \\ \langle \mathrm{M} \rangle \\ \langle \mathrm{H} \rangle \\ \langle \mathrm{M} \rangle \end{array}$	VH: alt_0 H: alt_1, alt_3 M: alt_2, alt_4
5	$\begin{array}{l} alt_0 = \langle \mathbf{H} \; \mathbf{M} \; \mathbf{H} \; \mathbf{V} \mathbf{H} \rangle \\ alt_1 = \langle \mathbf{H} \; \mathbf{VL} \; \mathbf{H} \; \mathbf{QL} \; \mathbf{QL} \rangle \\ alt_2 = \langle \mathbf{L} \; \mathbf{VL} \; \mathbf{N} \; \mathbf{H} \; \mathbf{QL} \rangle \\ alt_3 = \langle \mathbf{QL} \; \mathbf{H} \; \mathbf{M} \; \mathbf{L} \; \mathbf{VL} \rangle \\ alt_4 = \langle \mathbf{VL} \; \mathbf{N} \; \mathbf{L} \; \mathbf{M} \; \mathbf{QL} \rangle \end{array}$	W, as many as possible $W_5 = (.0, .0, .2, .4, .4)$ S_7 , non-symmetric $\frac{1}{1000} \frac{1}{1000} \frac{1}{1000}$	$\begin{array}{l} \langle \mathbf{H} \rangle \\ \langle \mathbf{M} \rangle \\ \langle \mathbf{M} \rangle \\ \langle \mathbf{M} \rangle \\ \langle \mathbf{L} \rangle \end{array}$	H: alt_0 M: alt_1, alt_2, alt_3 L: alt_4

Table 1. Aggregation-based results obtained in different scenarios

The results of applying the LOWA operator are given in the rating column of Table 1. We compare different situations by changing both the weighted vector W and the label set S. In all cases we obtain the same *best* alternative: alt_0 . If we observe the first column, this tuple has a lot of good rated labels, obtaining

a good rate at the end. In contrast, the worst alternative is alt_4 , because it has most of the attributes bad labelled. The rest of alternatives change the ranking position according to the conditions set by the parameters, because they have a mixture between good and bad labelled attributes.

5 Conclusions and future work

This paper presents a distributed patient-centered system that facilitates to the user the selection of appointments when a clinical test is required. We have explained how we combine (1) the use of a multi-agent system based on medical guidelines with (2) decision-making techniques that use two types of values, linguistic and numerical. The main part of the paper has been devoted to explaining the rating and ranking of the appointments. We proposed the use of the LOWA operator that uses fuzzy linguistic variables.

We are now busy designing a method to monitor the behavior of the patient (which of the proposed appointments is selected) in order to learn how the user's preferences evolve and improve the ranking over time. At the moment, the profile is updated by hand. Another future research line is the study of the adequacy of giving different weights to the attributes.

References

- Nealon, J.L., Moreno, A.: Agent-Based Applications in Health Care. In: Applications of Software Agent Technology in the Health Care Domain. Birkhäuser Verlag (2003) 3–18
- [2] Quaglini, S., Stefanelli, M., Cavallini, A., Micieli, G., Fassino, C., Mossa, C.: Guideline-based careflow systems. Artif Intell Med 20 (2000) 5–22
- [3] Seroussi, B., Bouaud, J., Chatellier, G.: Guideline-based modeling of therapeutic strategies in the case of chronic diseases. Int J Med Inform 74 (2005) 89–99
- [4] IOM: Clinical Practice Guidelines: Directions for a New Program. Institute of Medicine (IOM), National Academy Press, Washington, D.C (1990)
- [5] Quaglini, S., Ciccarese, P., Micieli, G., Cavallini, A.: Non-Compliance with Guidelines: Motivations and Consequences in a case study. In: Symposium on Computerized Guidelines and Protocols, CGP 2004, IOS Press (2004) 75–87
- [6] Herrera, F., Herrera-Viedma, E.: Linguistic decision analysis: steps for solving decision problems under linguistic information. Fuzzy Sets and Systems 115 (2000) 67–82
- [7] Isern, D., Moreno, A.: Distributed guideline-based health care system. In: Intel. Systems Design and Applications, ISDA-2004, IEEE Press (2004) 145–150
- [8] Peleg, M., Tu, S., et. al: Comparing Computer-Interpretable Guideline Models: A Case-Study Approach. J Am Med Inf Ass 10(1) (2003) 52–68
- [9] Ghinea, G., Magoulas, G.: Intelligent Protocol Adaptation in an Enhanced Medical e-Collaboration Environment. Int J Art Int Tools 13 (2004) 199–218
- [10] Delgado, M., Herrera, F., Herrera-Viedma, E., Martínez, L.: Combining numerical and linguistic information in group decision making. Inf Sci 107 (1998) 177–194
- [11] Grabisch, M., Orlovski, S., Yager, R.: Fuzzy aggregation of numerical preferences. In Slowinski, R., ed.: Fuzzy sets in decision analysis, operations research and statistics. Kluwer Academic (1999) 31–68
- [12] Yager, D.R.: On Ordered Weighted Averaging Aggregation. Operators in Multicriteria Decision making. IEEE Trans Syst Man Cybern 18 (1988) 183–190