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NEW RESULTS IN A SELF-ORGANIZED MODEL OF TECHNOLOGICAL EVOLUTION

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We present new results in a model of technological evolution which displays different macroscopic behaviors based on very simple microscopic rules of local interaction. The main features are criticality and self-organization. We give information about new scaling relation and study the roughness of the spatial technological profile. We verify that the performance is optimized in the critical region independently of the dynamical rules.

Keywords: Self-organized criticality, econophysics, socio-economic evolution.

1. Introduction

One of the most appealing examples of productive and fruitful collaboration between two different disciplines that can be found nowadays in science is econophysics. Physicist are providing mathematical tools and new insights to tackle problems that economists were eager to consider and as a consequence the new field is being developed at an unusual speed [2].[a]

Two basic branches are the main focus of interest. Perhaps the most important is finance. The analysis of the statistical properties of real time data (stock market), the control of financial risk or indication about to handle an optimal portfolio

[a]Since this is a fast growing field we should point the reader to the Internet pages devoted to the Econophysics literature `http://www.unifr.ch/econophysics/`, where an exhaustive list of references is permanently updated.

are some typical examples which are receiving a lot of attention from the physics community [5, 9, 7]. There is another group of people whose main goal is to develop agent based models with the hope that the analysis of the essential mechanisms governing the individual behavior of an ensemble of interacting units could give information about macroscopic observables that can be measured in real life [3]. Indeed, economic systems can display very complex cooperative behavior even if the individual nature of the agents is very simple. Agent models for trading, for companies growth [8] or for spatial dependent games are, among others, examples which have been investigated recently.

Coordination and competition are two basic ingredients that define the character of any agent based model. Competition is more suitable to consider evolutionary scenarios where the amount of resources is limited. Coordination fits well with the idea of growth and optimization of common resources. Recently, we have studied a model for technological progress where coordination and cooperation between agents give rise to interesting features that can be observed in real economies [1]. The model, which will be sketched briefly in the next section, displays self-organization, scale-dependent effects (larger economies grow faster than smaller ones) and allows to define observables that quantify the rate of progress of the system.

In this paper we present new results that not only will help to understand better the collective behavior of the model but also other important aspects such that fluctuations or scaling properties.

2. The Model

We have defined a model with local couplings formed by n agents, each of them occupying a particular position (node) in a distributed one-dimensional boundaryless lattice (see Fig. 1). At every time step (time will be considered discrete) each agent adopts a certain action $a_i(t)$ that may be interpreted as the technology level it currently uses. From the interaction with its first nearest neighbors (left and right) it obtains corresponding payoffs, $\psi(a_i(t), a_{i+1}(t))$ and $\psi(a_i(t), a_{i-1}(t))$, with $\psi(\cdot)$ being called the *payoff function.*

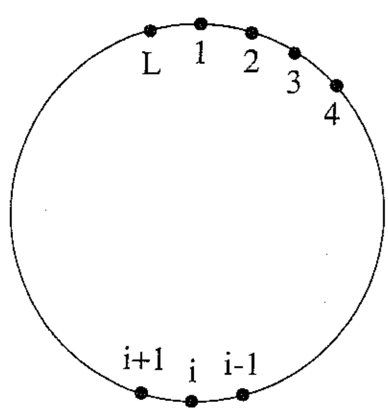

Fig. 1. Distribution of the agents n a one-dimensional ring.

The interaction must reflect a crucial aspect of the model: the compatibility of technological levels should lead to higher payoffs while some incompatibility costs that detract from the base payoff, should arise from a possible mismatch between neighbors' technologies. In addition, it seems logical to assume that the costs must be bounded from below (the bankrupt). As an example, we may think of the interaction between two agents as consisting of the completion of a new project (e.g. launching a new product through firm partnership), for which dissimilarity of action leads to consequent waste of resources. Under these general requirements we have chosen the following payoff function

$$\psi(a,a') = \begin{cases} a - k_1\left(1 - e^{-(a-a')}\right) & \text{if } a \geq a' , \\ a - k_2\left(1 - e^{-(a'-a)}\right) & \text{if } a < a' . \end{cases} \qquad (1)$$

The base payoff obtained from using a certain technology is assumed equal to a while the incompatibility costs due to being too advanced or too backwards relative to the neighbors are parameterized, respectively, by positive factors k_1 and k_2.

To model a dynamic environment in which infrequent perturbations punctuate periods of stasis, thus triggering relatively quick processes of diffusion, we propose an adjustment dynamics decoupling both processes as follows (see Fig. 2):

- Updates (marked (i) in Fig. 2): at each time step a randomly selected agent is chosen to update its technological level from $a_i(t-1)$ to

$$a_i(t) = a_i(t-1) + \tilde{\sigma} , \qquad (2)$$

where $\tilde{\sigma}$ is an i.i.d random variable. This update plays the role of an exogenous perturbation that may be attributed to several interpretations (e.g. local innovation, a shock in payoffs, population renewal, etc.).

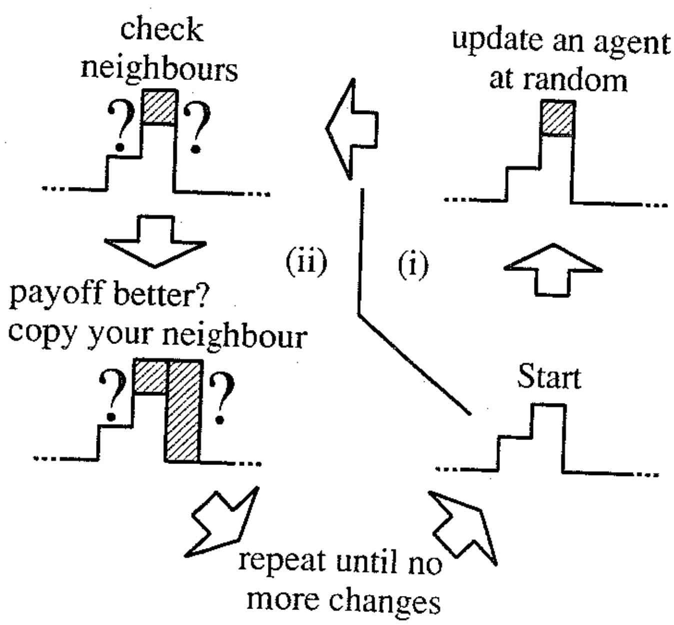

Fig. 2. Schematic description of the dynamics of the model. The two time-scales are shown; (i) is the slow time-scale (innovation) and (ii) is the fast time-scale (diffusion).

- Diffusion (marked (ii) in Fig. 2): each one of agents $i+1$ and $i-1$ now have three options: to maintain its level or adopt one of its two neighbors. It will choose the one which maximises its payoff i.e. $\psi(a_i, a_{i-1}) + \psi(a_i, a_{i+1})$. This process continues until no agent wants to perform any adjustment in its technological level. Then, another agent is updated randomly and so on.

Notice, that both processes are defined in different time scales. While the innovations are supposed to be slow, the diffusion of them is a very fast process. The diffusion is completely resolved before any new update is done so that the model is synchronous.[b] For each "event" taking place at time t, i.e. a random update followed by the upgrades it induces, we define the *event size* $s(t)$ as the number of agents that were modified, and the technological advance $H(t)$ as the sum of the technology gain throughout the system.

In our previous work [1] it turned out that the key to understand the dynamics of the system relays only in the difference $k \equiv k_1 - k_2$. Intuitively, k reflects the cost difference resulting from "downwards incompatibility" (i.e. being too advanced) as compared to that derived from "upwards incompatibility" (i.e. being too backwards). If k is small then all the agents tend to have the best possible technology and any new update will immediately be copied by the whole population. In the fast time scale it means a wave of change (avalanche) involving a large number of agents in such a way that after each update (slow time scale) the agents are synchronised i.e. have the same a. We will refer to this regime as the *synchronised growing front* regime since when the technological profile of the system is considered, as we will discuss in Sec. 4, this regime is characterised by a flat profile advancing with time. In the limit of large k it is difficult to find an agent interested in changing their current state because the cost is too high, so that the avalanches are very small. We may identify both extremes as a supercritical and sub-critical state, respectively. The intermediate range is very rich and the most interesting from a dynamic standpoint.

3. Scaling Properties

As pointed out in the last section, for a finite range of values of the parameter k, the dynamics of the model produces events of all sizes. This scale invariance is the hallmark of self-organized critical behavior, confirmed by the power-law distributions for different magnitudes obtained in numerical simulations [1]. In Fig. 3 we have represented the distribution of event sizes for different total number of agents L. We observe a fast decay for small event sizes followed by a power-law behavior. The picture also shows that the initial decay is independent of the system size.

In order to make more quantitative assertions about the concrete form that this distribution adopts in our model, we propose the following method.

[b]An asynchronous model where the update and diffusion can overlap was also investigated but the results are similar. This variant will not be discussed here.

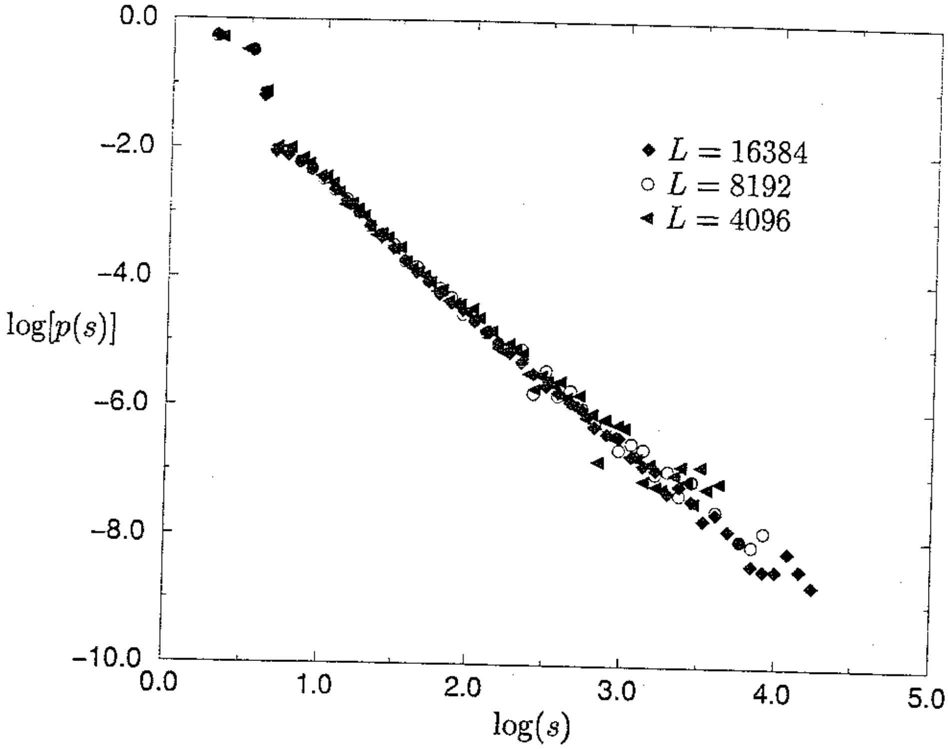

Fig. 3. Event size distribution for different system sizes and $k = 4$.

We calculate the probability, denoted P, that an agent adjusts its technological level (upgrade) after an innovation (update) has been introduced in the system. We can relate P to the event size distribution, $p(s)$, if we interpret it as the probability that an update generates an avalanche of size s. Given this, our agent will be affected with probability s/L, being L the size of the system. Assuming for $p(s)$ a continuous form, we can express P as

$$P = \int_0^L p(s) \frac{s}{L} \, ds = \langle s \rangle / L \,. \tag{3}$$

Now it is a simple exercise to obtain P, given a concrete form for the distribution of event sizes. Based on Fig. 3 we propose to approximate the event size distribution by the following continuous distribution,

$$p(s) = \begin{cases} C_1 e^{-s} & \text{if } s \in [0, s_0)\,, \\ C_2 s^{-\gamma} & \text{if } s \in [s_0, L] \end{cases} \tag{4}$$

assuming an exponential decay for event sizes up to s_0 and a power-law for larger event sizes. The continuity condition between the different parts of the distribution relates C_1 and C_2,

$$C_1 e^{-s_0} = C_2 {s_0}^{-\gamma}\,, \tag{5}$$

and the normalization condition enables to determine C_2,

$$C_2 = {s_0}^{\gamma} \left(e^{s_0} - 1 + \frac{s_0}{1-\gamma} \left[\left(\frac{L}{s_0} \right)^{1-\gamma} - 1 \right] \right)^{-1} . \tag{6}$$

Now it is a straightforward calculation to obtain P as a function of γ and s_0. For $\gamma \neq 2$ we have

$$P(\gamma, s_0) = \frac{1}{L} \times \frac{e^{s_0} - (s_0+1) + \frac{s_0{}^2}{2-\gamma}\left[\left(\frac{L}{s_0}\right)^{2-\gamma} - 1\right]}{e^{s_0} - 1 + \frac{s_0}{1-\gamma}\left[\left(\frac{L}{s_0}\right)^{1-\gamma} - 1\right]}, \tag{7}$$

whereas for $\gamma = 2$ the expression is

$$P(\gamma = 2, s_0) = \frac{1}{L} \times \frac{e^{s_0} - (s_0+1) + s_0{}^2 \ln\left(\frac{L}{s_0}\right)}{e^{s_0} - 1 - s_0\left[\frac{s_0}{L} - 1\right]}. \tag{8}$$

Given the relationship just obtained between the probability that an agent makes an upgrade and the parameters characterizing the event size distribution, the numerical estimation of P allows us to say something about the form of this distribution. To estimate P we have to collect the number of upgrades per event that the agents make. In Fig. 4 we have represented the distributions for a system of size $L = 4096$ and three different values of k. The mean values of these distributions provide an estimation of P. To compare with the analytical results we have used the values of the exponent γ obtained by linear regression. In Fig. 5 we have represented these points together with the analytical results. We observe a good fit with the line corresponding to the case $s_0 = 3$, confirming that the rapid decay occurs, indeed, for very small event sizes.

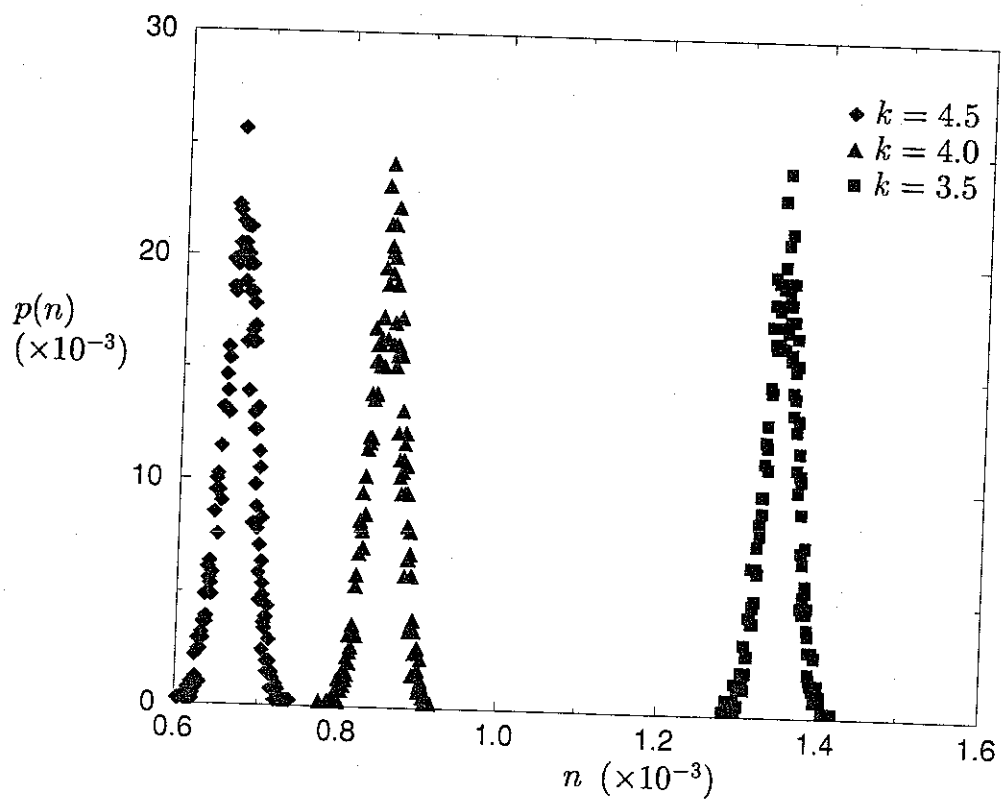

Fig. 4. Probability distribution of the number n of upgrades per event and per agent for $k = 4.5$ (◆), $k = 4.0$ (▲), and $k = 3.5$ (■), and a sample of size 4096.

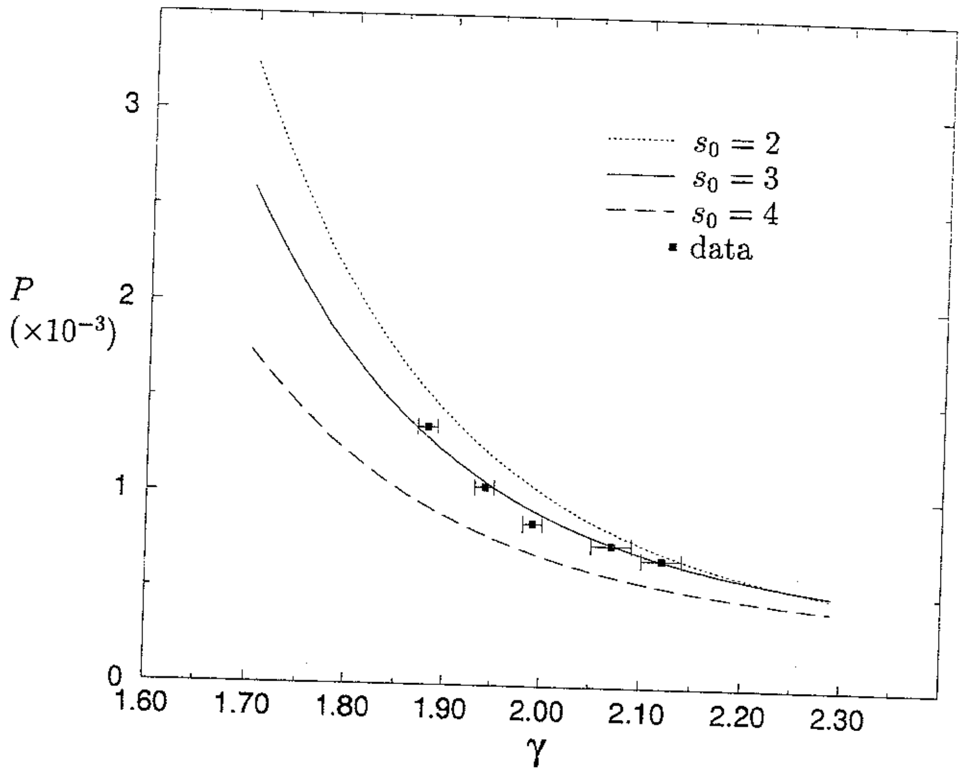

Fig. 5. Comparison between analytical results and the data obtained in simulations. The lines correspond to evaluating P as a function of γ and s_0 (Eqs. (7)–(8)), and the squares to the pairs of values of P and γ obtained from the simulation for different values of k.

4. Profiles

In this section, we are going to focus our attention on the geometric properties of the technological profile that results from the evolution of the system. In this way, we will be able to extract some information about the spatial fluctuations of the system. The technology profile, defined by the technological level of the agents a_i, exhibits several regimes. For $k < 3$, the profiles are essentially flat since big avalanches are very frequent and they tend to flatten the interface. On the other hand, for $k > 3$ the profiles, as Fig. 6 shows, are quite rough. In fact, the larger the value of k the more frequent small avalanches are and the rougher the interface is. In this work, we will analyze technological profiles with k close to the critical region $k \sim 3.5$ since this is the most interesting regime. Beginning from a flat initial interface, fluctuations statistically increase each time step. Nevertheless, fluctuations do not grow forever, they saturate after some simulation steps. Moreover, the larger the system size the later they saturate.

Given this phenomenology, one can treat the system as a non-equilibrium interface and try to apply some methods used in surface growth problems [4]. In such problems it is common to study the fluctuations of the global interface width which is defined as

$$W(L,t) \equiv \left\langle \frac{1}{L} \sum_i \left[a_i(t) - \overline{a_i(t)}\right]^2 \right\rangle^{1/2}, \qquad (9)$$

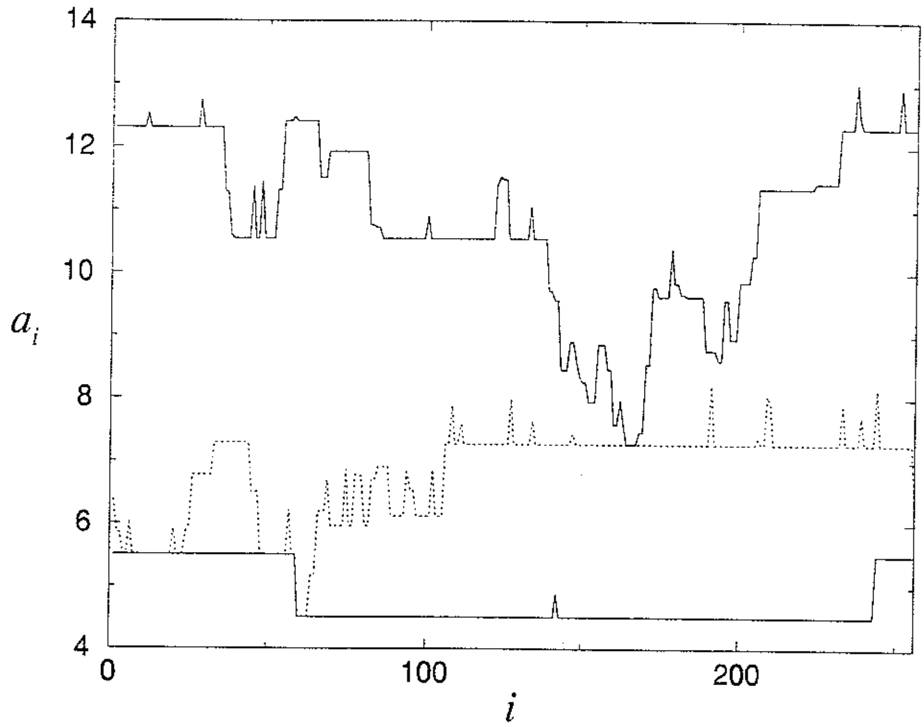

Fig. 6. Three snapshots of the technological interface at different times for a system of 256 agents and $k = 4.5$. The interface width fluctuations grow with time until they reach a statistically stationary value.

where $\langle \cdots \rangle$ implies averaging over different realizations and $\overline{a_i(t)}$ is the average technological level at time t. Once the interface width has saturated, it is possible to calculate the *roughness exponent* α by means of the scaling law

$$W_{\text{sat}}(L) \sim L^{\alpha} . \tag{10}$$

If this relation holds, then the interface is said to be self-similar in the stationary state. Sometimes is more convenient to work with the local interface width, $w(l,t)$. It is defined in analogy with Eq. (9), but now the spatial average is performed over a window of size l instead of over the whole interface L. In Fig. 7 the evolution of $w(l,t)$ for some l are shown. The local interface width is related to the height-height correlation function $G(l,t) = \langle \overline{(a_{i+l}(t) - a_i(t))^2} \rangle$ through the scaling relation $w(l,t)^2 \sim G(l,t)$. Furthermore, given a system size L, we can estimate the roughness exponent by calculating $w_{\text{sat}}(l)$ for several window sizes l and assuming that

$$w_{\text{sat}}(l) \sim l^{\alpha} . \tag{11}$$

This is true for self-similar interfaces, but not in general. In particular, there are some interfaces that show *anomalous scaling* [6], that is, local fluctuations do not scale as global fluctuations. In this case, a new exponent is introduced, the *local roughness exponent* α_{loc}, to characterize the local fluctuations. Now Eq. (11) reads

$$w_{\text{sat}}(l) \sim \sqrt{G_{\text{sat}}(l)} \sim l^{\alpha_{\text{loc}}} L^{\alpha - \alpha_{\text{loc}}} . \tag{12}$$

Notice that, for $l = L$, Eq. (10) is recovered. Our model exhibits an anomalous scaling in the stationary state for k close to the critical region. For $k = 3.5$ we

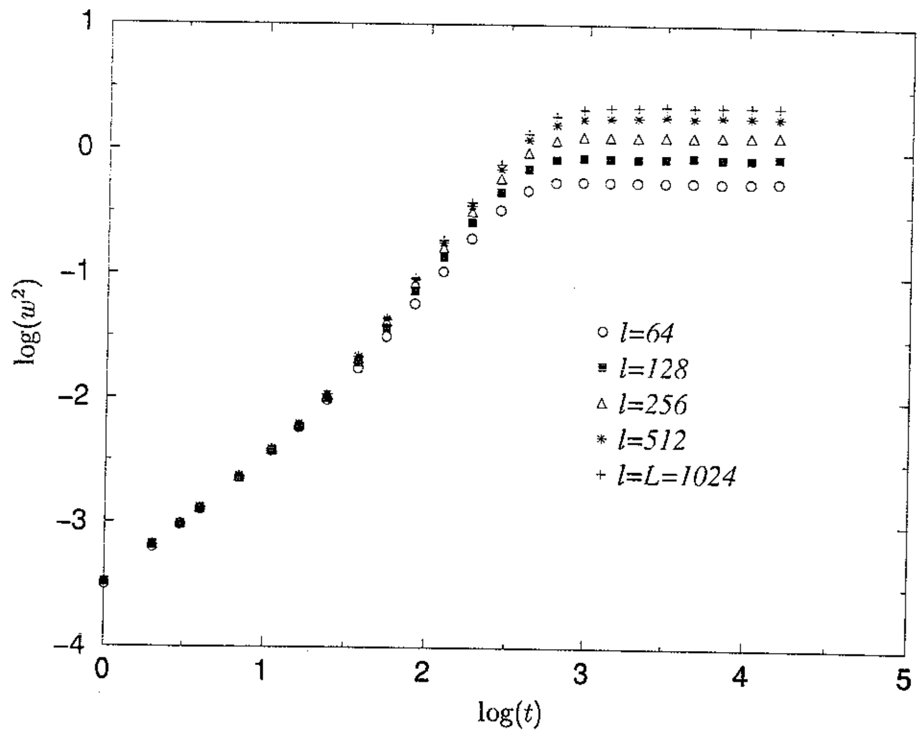

Fig. 7. Time evolution of the fluctuations of the local interface width $w(l,t)$ for different window sizes l up to system size $L = 1024$. They saturate at a value $w_{\text{sat}}(l)$ which increases with window size l.

obtain roughness exponents $\alpha \sim 0.18$ and $\alpha_{\text{loc}} \sim 0.34$. Therefore, in the model, global economic fluctuations, do not scale in the same way as local fluctuations do. These results only concern the spatial scaling of fluctuations in the stationary state. The transient, or temporal, scaling of the interface does not seem to fit any of the known scaling families and is currently being investigated.

5. Introducing Costs

Another ingredient with respect to the original model could be introduced. This concerns the cost an agent should pay when updating to a higher technological level along a diffusion process. This means that when an agent takes a decision it is based not only on the difference in payoffs, as explained in Sec. 2, but also on a cost that in general can depend on the amount of the upgrade, $C(\Delta)$. In other words, it chooses within the set $\{a_{i-1}, a_i, a_{i+1}\}$

$$\begin{aligned} a_i \to a_{i-1} &: \Psi(a_{i-1}, a_{i-1}) + \Psi(a_{i-1}, a_{i+1}) - C(a_{i-1} - a_i)\,, \\ a_i \to a_i \quad &: \Psi(a_i, a_{i-1}) + \Psi(a_i, a_{i+1})\,, \\ a_i \to a_{i+1} &: \Psi(a_{i+1}, a_{i-1}) + \Psi(a_{i+1}, a_{i+1}) - C(a_{i+1} - a_i) \end{aligned} \tag{13}$$

and takes the one that maximizes the RHS of (13).

The main effect of this introduction is to disable very small jumps. Let us see, for instance, how the condition for stability of the synchronized growing front, defined in the last paragraph of Sec. 2 and studied in [1], changes. Imagine a flat profile

$a_{i-1} = a_i = a_{i+1}$, and then update $a_{i-1} \to a_{i-1} + \Delta$. Now agent i will decide to upgrade if the following condition is satisfied

$$k_1 - k_2 < \frac{2\Delta - C(\Delta)}{1 - e^{-\Delta}}. \tag{14}$$

Since we have assumed a continuous distribution of technological levels, the strongest condition corresponds to $\Delta \to 0$. Then it is easy to see that if $\lim_{\Delta \to 0} C(\Delta) \neq 0$, i.e. even a very small update has a finite cost, then the condition of stability of the synchronized front is $k_1 - k_2 \to -\infty$. This means that the uniformly growing front is no longer a possible dynamics of the system. Anyway, big avalanches are still possible if local gradients are large enough.

We have performed computer simulations with a fixed cost of 0.5 units per upgrade and random updates $\tilde{\sigma}$ (as defined in Eq. (2)) within the interval [0, 1]. For small values of k, even smaller than 2, we observe very large avalanches but not all of them are system size wide. Hence, this mechanism provides an improvement in the efficiency of the system. In [1] we defined the technological advance rate as

$$\rho = \lim_{T \to \infty} \rho(T) = \lim_{T \to \infty} \frac{\sum_{t=1}^{T} H(t)}{\sum_{t=1}^{T} s(t)}. \tag{15}$$

The values of ρ obtained from simulations with the new dynamical rules are plotted in Fig. 8. Now it is easy to realize that since small upgrades are forbidden

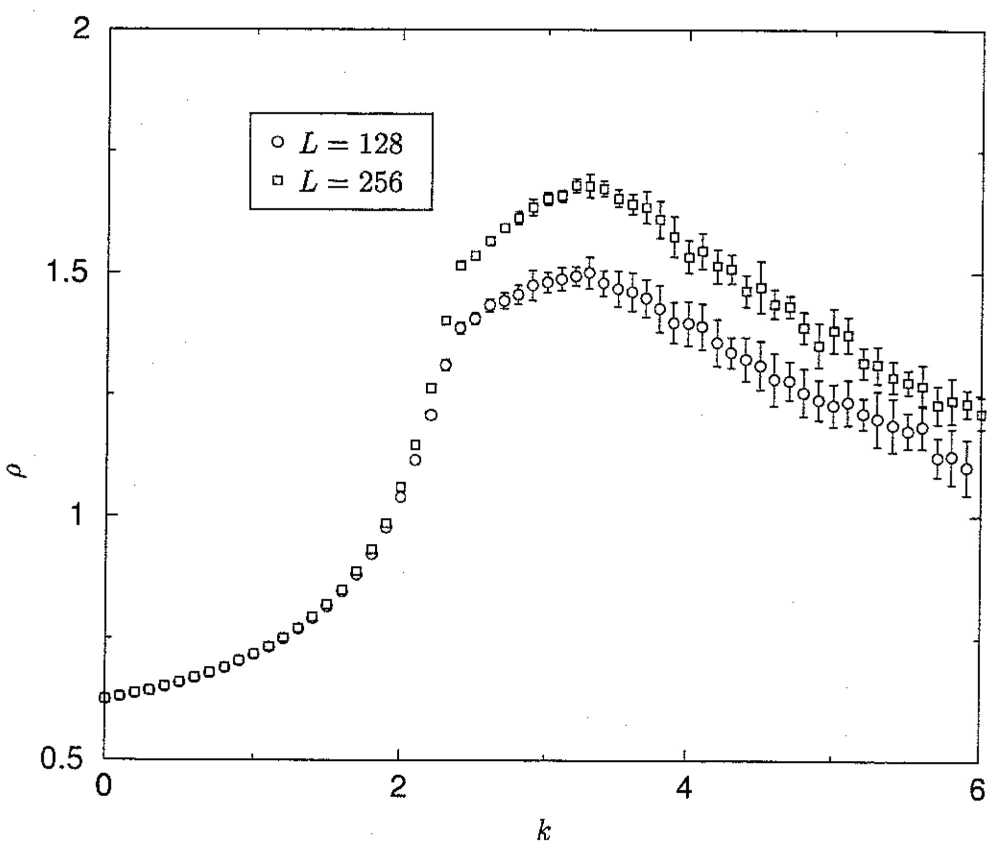

Fig. 8. Efficiency as a function of k, for two different system sizes.

the system will be able to reach a global technological level with fewer upgrades and hence the parameter ρ is larger, for a given value of k, than in the original model.

For larger values of k (bigger than 3) we enter into the critical region where avalanches are power-law distributed. With respect to the original model we expect slightly better values of ρ, 25% for $k = 3$ and 2% for $k = 4$ for instance, but different numerical values of the exponents. These two effects are, nevertheless, related since the maximum efficiency is still within the critical region and it grows with system size, showing again interesting scale effects.

Finally, for very large values of k the effect of introducing costs is negligible since in this case upgrades are already quite large.

6. Conclusions

We have presented new results in the study of evolution in socio-economic environments, with special attention on the subject of technological progress. Following the trends defined in a previous work we have confirmed that this model presents some of the features characterizing self-organized critical systems in the framework of statistical physics. For instance, we have found a new way of computing the exponents of the power law distribution of event sizes. Borrowing some ideas from interface growing phenomena we have studied the roughening of the technological profile, observing that it has an anomalous scaling, which can be quite interesting from an economic point of view, since local and global fluctuations behave in a different way. Finally, the concept of universality shows up when changing the microscopic rules but optimization of the magnitude that characterizes efficiency still takes place within the critical region; the numerical values of the exponent change, the width of the critical region enlarges, but these facts do not change the location of the efficiency peak.

Acknowledgment

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