

# Identification of boundary planes in three-dimensional flows

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## Abstract

Let  $\vec{v} = \vec{v}(p, t)$  be the velocity field of a Newtonian fluid,  $\vec{\omega} = \vec{\omega}(p, t)$  its vorticity field and  $(e_{ij})$  its 2-covariant rate-of-strain tensor. In this paper we give a formulation to identify boundary planes in analytical and numerical three-dimensional flow fields. The proposed formulation is based on the calculation of the locus where  $\sum_{i,j=1}^3 v_i \omega_j e_{ij} = 0$  is verified.

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## 1. Introduction

Several authors have proposed techniques and definitions to detect vortical structures in flows. This kind of flow organization is of importance to analyze the structure and the properties of the flow because vortices are commonly responsible for mixing processes and for large rates of momentum and heat/mass transfer (see, for example, Hunt et al. [1], Chong et al. [2], Jeon and Hussain [3], Michard et al. [4], Cucitore et al. [5], Wu et al. [6], Haller [7], Roth and Peikert [8,9]).

All these techniques, although useful to study the structure and organization of flows, have virtues and problems. In this study we report another method to extract information of the flows proposing an analytic procedure to identify boundary planes in three-dimensional flow fields. We also present some examples to illustrate the proposed technique.

## 2. Boundary plane

Let be  $\mathcal{F}$  a flow of a Newtonian fluid in  $\mathbb{R}^3$  (oriented Euclidean space of dimension three), then we can consider the trio  $(\vec{v}(p, t), \vec{\omega}(p, t), D_{(p,t)})$  formed by the smooth velocity vector field of  $\mathcal{F}$ , its vorticity field;  $\text{curl}(\vec{v})$ ; and its 2-covariant rate-of-strain tensor, respectively.

We designate a boundary plane of  $\mathcal{F}$  a plane  $\sigma$  such that it separates regions of the flow without momentum exchange. Particularly, the properties of this plane can be stated as:

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