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The conics of Lucas' configuration

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1 Introduction

Let us consider a figure formed by a triangle ABC and its three inscribed squares $X_1X_2Y_3Z_4$, $Y_1Y_2Z_3X_4$, $Z_1Z_2X_3Y_4$, where the sides X_1X_2, Y_1Y_2, Z_1Z_2 are on the sides AB, BC, CA of the triangle, and these three squares are homothetic to the external squares $BAB'A'$, $CBC'B'$, $ACA'C'$, respectively, from the vertices of CAB; see Fig. 1. We will call this figure "Lucas' configuration".

In fact, there are another three squares inscribed in the triangle ABC. These are the three squares $X'_1X'_2Z'_3Y'_4$, $Y'_1Y'_2X'_3Z'_4$, $Z'_1Z'_2Y'_3X'_4$, where the sides $X'_1X'_2$, $Y'_1Y'_2$, $Z'_1Z'_2$ are on the sides $A'B'$, $B'C'$, $C'A'$ of the triangle, and these three squares are homothetic to the internal squares $ABA''B''$, $BCB''C''$, $CAC''A''$, respectively, from the vertices of CAB. We will call this figure "Lucas' internal configuration"; but the results and conditions are similar to Lucas' configuration.

In [3], I. Panakis shows the relations found by Édouard Lucas between the circumcircles of the triangles AX_4Z_3, BY_4X_3, CZ_4Y_3 and the length of the sides of the triangle ABC. In [1], A.P. Hatzipolakis and P. Yiu show that these three circumcircles are mutually tangent to each other, and tangent to the circumcircle; see Fig. 1.

In this note we show that Lucas' configuration has more geometric peculiarities. We find the following result:

Der vorliegende Beitrag ist eine Variation zur sogenannten Lucas-Konfiguration. Diese ist beschrieben durch ein Dreieck und die ihm einbeschriebenen drei Quadrate, deren eine Seite jeweils auf einer der Dreiecksseiten liegt. Der Autor beweist nun das bemerkenswerte Resultat, dass die zwölf Eckpunkte der drei Quadrate in zwei Klassen mit je sechs Punkten zerfallen, so dass die Punkte beider Klassen jeweils einen Kegelschnitt beschreiben. Eine Klasse beschreibt dabei sogar eine Ellipse.