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The vortical deviation on a stream surface

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ABSTRACT

Let S be a stream surface in a flow. Let ω_3 be the vertical component of the vorticity on S. In the present work we make an extension of the concept of the vorticity on S; we define the *geodesic vorticity* Ω and the *vortical deviation* $\mathcal{D}$ and we present some of its properties. The geodesic vorticity Ω will be $\Omega = \frac{\partial u_2}{\partial s_1} - \frac{\partial u_1}{\partial s_2}$ where $\overrightarrow{u} = (u_1, u_2, 0)$ is the velocity field on S, and s_1, s_2 are, respectively, the arc length parameters of the lines of the maximum and minimum normal curvature on the surface S. The vortical deviation $\mathcal{D}$ will be the difference between the geodesic vorticity Ω and the vorticity ω_3, that is $\mathcal{D} = \Omega - \omega_3$. The main results of this work are the relation between $\mathcal{D}$ and the curvatures on S (total curvature, geodesic curvature).

1. Introduction

The study of the vorticity on interfacial surfaces is of considerable interest. Recent works relate the geometry of the surfaces with the components of the vorticity—for example Wu [1], and Dopazo, Lozano and Barreras [2]. For S a steady and free surface (both tangential components of the stress tensor vanish at the surface), Longuet-Higgins [3] shows the relationship between the tangential components of the vorticity field on S and the normal curvatures of S. In Herrera [4] we consider the case where S is a stream surface, and we find the relationship between the three components of the vorticity field on S and the curvatures of the streamlines (geodesic torsion, normal curvature and geodesic curvature).

We consider a stream surface S in a flow; that is: let S be a $\mathcal{C}^\infty$-smooth surface of $\mathbb{R}^3$ (oriented Euclidean three-dimensional space) tangent to the $\mathcal{C}^\infty$-smooth velocity vector field $\overrightarrow{u} = \overrightarrow{u}(p, t)$ of the flow at any fixed time $t = t_0$.

Let $\overrightarrow{x}$ be a parametrization of the smooth surface S:

$$\begin{aligned} \overrightarrow{x} : U \subset \mathbb{R}^2 &\longrightarrow \mathbb{R}^3 \\ (\xi_1, \xi_2) &\longrightarrow \overrightarrow{x}(\xi_1, \xi_2) = (x(\xi_1, \xi_2), y(\xi_1, \xi_2), z(\xi_1, \xi_2)) . \end{aligned}$$

We consider the velocity vector field $\overrightarrow{u} = \overrightarrow{u}(p, t)$ and the vorticity vector field $\overrightarrow{\omega}(p, t) = \text{curl}(\overrightarrow{u})$ on the stream surface S.

In the present work we make an extension of the concept of the vorticity on the stream surface S; we define the *geodesic vorticity* Ω, we define the *vortical deviation* $\mathcal{D}$ and we present its properties.

If S is a plane, we can consider Cartesian coordinates (x, y, z) in $\mathbb{R}^3$ such that $z = 0$ is the equation of S. With these coordinates we can write the velocity vector field as $\overrightarrow{u} = (u_1, u_2, u_3)$ and the vorticity vector field as $\overrightarrow{\omega} = (\omega_1, \omega_2, \omega_3)$. Then we have that $u_3 = 0$ and that ω_3 is the orthogonal component of $\overrightarrow{\omega}$ on S. It is well-known that

$$\omega_3 = \frac{\partial u_2}{\partial x} - \frac{\partial u_1}{\partial y}.$$

We can observe, trivially, that $\gamma_a(x) = (x, k_1, 0)$, $\gamma_b(y) = (k_2, y, 0)$, with k_1, k_2 constants, are orthogonal lines of the maximum and minimum normal curvature respectively on the surface S (that is lines of curvature of S). Also we can observe that x and y are respectively the arc length parameters of these lines.

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