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NOTE ABOUT THE ISOCHRONICITY OF HAMILTONIAN SYSTEMS AND THE CURVATURE OF THE ENERGY

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Abstract. In this note we show the relationship between the period of a isochronous center of a planar Hamiltonian system and the Gauss curvature of the surface $S = (x, y, H(x, y))$ where H is the energy function of the system.

1. Introduction

It is well known that the planar analytic Hamiltonian systems are the planar differential systems of the form

$$\begin{cases} \dot{x} = -\frac{\partial H}{\partial y}(x,y) \\ \dot{y} = \frac{\partial H}{\partial x}(x,y), \end{cases} \tag{1.1}$$

where H is an analytic function on $\mathbb{R}^2$. The solutions of these systems are contained in the level curves $\{H(x,y) = h,\ h \in \mathbb{R}\}$. A point p is called center if it has a neighborhood formed by periodic orbits. The largest neighborhood of p which is entirely covered by periodic orbits is called the *period annulus* of p and we will denote it by $\mathcal{P}$. The function which associates to any periodic orbit γ in $\mathcal{P}$ its period is called the *period function.* The center is called *isochronous center* when the period function is a constant ω. It is well known that only nondegenerate centers can be isochronous. And from now on we will assume that $H(0,0) = 0$ and the system (1.1) has a nondegenerate center at the origin.

Cima, Mañosas and Villadelprat in [1] study the isochronous centers of Hamiltonians systems of the form

$$H(x,y) = A(x) + B(x)y + C(x)y^2 \tag{1.2}$$

with A, B, C analytical functions. They prove that if the center is isochronous of period ω then

$$\frac{d^2\left(4AC - B^2\right)}{dx^2}(0) = \frac{8\pi^2}{\omega^2}. \tag{1.3}$$

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