

## The transverse structure of Lie flows of codimension 3

By

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### 1. Introduction

This paper deals with the problem of the realization of a given Lie algebra as transverse algebra to a Lie foliation on a compact manifold.

Lie foliations have been studied by several authors ([E.H.S], [E.N], [F], [H.M], [M], [Ma], etc.). The importance of this study was increased by the fact that they arise naturally in Molino's classification of Riemannian foliations [M].

To each Lie foliation are associated two Lie algebras, the Lie algebra  $\mathcal{G}$  of the Lie group on which the foliation is modeled and the structural Lie algebra  $\mathcal{H}$ . The latter algebra is the Lie algebra of the Lie foliation  $\mathcal{F}$  restricted to the closure of any one of its leaves. In particular, it is a subalgebra of  $\mathcal{G}$ . We remark that although  $\mathcal{H}$  is canonically associated to  $\mathcal{F}$ ,  $\mathcal{G}$  is not.

Thus two interesting problems are naturally posed: the *realization problem* and the *change problem*.

The *realization problem* is to know which pairs of Lie algebras  $(\mathcal{G}, \mathcal{H})$ , with  $\mathcal{H}$  subalgebra of  $\mathcal{G}$ , can arise as transverse and structural Lie algebras, respectively, of a Lie foliation  $\mathcal{F}$  on a compact oriented manifold  $M$ .

This problem is closely related to the following Haefliger's problem [Ha]: given a Lie subgroup  $\Gamma$  of a Lie group  $G$ , is there a Lie  $G$ -foliation on a compact manifold  $M$  with holonomy group  $\Gamma$ ? E. Ghys [Gh] and G. Meigniez [Mg] also studied this problem and they gave necessary conditions for a pair  $(G, \Gamma)$  to be realizable.

Our formulation of the realization problem is a little different: We shall say that the pair  $(\mathcal{G}, q)$  is *realizable* if there is a compact oriented manifold endowed with a Lie foliation transversely modeled on  $\mathcal{G}$  and with structural Lie algebra of dimension  $q$ . We also say that  $\mathcal{G}$  is realizable as *transverse* to a Lie foliation.

This formulation of the *realization problem* has been considered in [L1], [H], [G, R] and [H.Ll.R] making a very detailed study of Lie flows of codimension 3 (cf. §8). But a complete classification was not obtained because of the following open question: