

## Transverse structure of Lie foliations

By Blas HERRERA, Miquel LLABRÉS  
and Agustí REVENTÓS

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### 0. Introduction.

This paper deals with the problem of the realization of a given Lie algebra as transverse algebra to a Lie foliation on a compact manifold.

Lie foliations have been studied by several authors (cf. [4], [5], [6], [12], [17]). The importance of this study was increased by the fact that they arise naturally in Molino's classification of Riemannian foliations (cf. [14]).

To each Lie foliation are associated two Lie algebras, the Lie algebra  $\mathcal{G}$  of the Lie group on which the foliation is modeled and the structural Lie algebra  $\mathcal{H}$ . The latter algebra is the Lie algebra of the Lie foliation  $\mathcal{F}$  restricted to the closure of any one of its leaves. In particular, it is a subalgebra of  $\mathcal{G}$ . We remark that  $\mathcal{H}$  is canonically associated to  $\mathcal{F}$ , but  $\mathcal{G}$  is not.

Thus two interesting problems are naturally posed: the *realization problem* and the *change problem*.

The *realization problem* is to know which pair of Lie algebras  $(\mathcal{G}, \mathcal{H})$ , with  $\mathcal{H}$  a subalgebra of  $\mathcal{G}$ , can arise as transverse and structural Lie algebras, respectively, of a Lie foliation  $\mathcal{F}$  on a compact manifold  $M$ .

This problem is closely related to the following Haefliger's problem (see [9]): given a subgroup  $\Gamma$  of a Lie group  $G$ , is there a Lie  $G$ -foliation on a compact manifold  $M$  with holonomy group  $\Gamma$ ?

The present formulation of the *realization problem* in terms of Lie algebras was first considered in [10], and [7] made a very detailed study of Lie flows of codimension 3. But a complete classification was not obtained because of the following open questions:

- i) Let  $\mathcal{G}_7^k$  be the family of Lie algebras for which there is a basis  $\{e_1, e_2, e_3\}$  such that

$$[e_1, e_2] = 0, \quad [e_1, e_3] = e_1, \quad [e_2, e_3] = ke_2, \quad k \in [-1, 0) \cup (0, 1].$$

For which  $k$  is there a Lie  $\mathcal{G}_7^k$ -flow on a compact manifold with basic