

EIGHT-CENTERED OVAL QUASI-EQUIVALENT TO KEPLER'S ELLIPSE OF PLANETS' TRAJECTORY

ÓVALO DE OCHO CENTROS QUASI-EQUIVALENTE A LA ELIPSE DE KEPLER DE LA TRAYECTORIA DE LOS PLANETAS

B. HERRERA^{a†} y A. SAMPER^b

a) Departament d'Enginyeria Informàtica i Matemàtiques, Universitat Rovira i Virgili, Spain; blas.herrera@urv.net[†]

b) Unitat predepartamental d'Arquitectura, Universitat Rovira i Virgili, Spain.

[†] corresponding author

Recibido 9/5/2016; Aceptado 3/6/2016

This paper has academic nature, nevertheless it is an application from [9] that we think can be interesting to students and instructors of undergraduate level of physics. With low level of geometric and computational techniques the readers can reproduce the results of the present paper. The most important thing is that these results clearly show that it is not enough take numerical measures of the orbits of the bodies to decide which are the real equations and laws that govern them. We present an approximation of the Kepler's ellipse of the planets' trajectory by circular arcs, which is *quasi-equivalent*; that is: we present the approximation of the Kepler's ellipse \mathcal{E}_b by the eight-centered oval $O_{\mathcal{E}_b}$ having the same center, axes, vertices, perimeter length and curvature at the vertices as \mathcal{E}_b , and also having practically negligible difference with respect to the surface area of \mathcal{E}_b , and also having barely distinguishable deformation error in relation to \mathcal{E}_b .

Este artículo, aunque tiene carácter académico en su naturaleza y presentación, consiste en una aplicación de las conclusiones geométricas de [9] la cual consideramos puede ser interesante para estudiantes y profesores en licenciaturas y grados de física. Con bajo nivel de geometría y técnicas computacionales, los lectores pueden reproducir los cálculos del presente trabajo. El aspecto más importante que se presenta aquí es que los resultados muestran que no es suficiente tomar medidas numéricas de las órbitas de cuerpos para poder determinar las ecuaciones y leyes reales que las gobiernan. Más concretamente, presentamos una aproximación, por arcos circulares, de la elipse de Kepler de las trayectorias de los planetas, la cual es *quasi-equivalente*. O sea, presentamos la aproximación de la elipse de Kepler \mathcal{E}_b por el óvalo de ocho centros $O_{\mathcal{E}_b}$ el cual posee: mismo centro, ejes, vértices, perímetro, y curvatura en los vértices que \mathcal{E}_b , y además la diferencia entre las áreas de \mathcal{E}_b y $O_{\mathcal{E}_b}$ es prácticamente nula y el error de deformación entre ambos es por añadidura prácticamente indistinguible.

PACS: 01.40.Fk Physics education, 01.50.H- Computers in education, 02.40.Dr Euclidean geometries, 95.10.Eg Orbit determination.

Approximating ellipses by circular arcs has been a classic subject of study by geometers. This has long been used for a wide range of applications, for instance in geometry, art, architecture. The reader can easily find a great deal of classical literature on these topics, in special for eight-centered ovals and four-centered ovals (also named *quadrarcs*). Moreover, because of its importance, this subject of study is continued in modern research papers as [1–7]. In astronomy this kind of approximation was classically considered, for example in [8].

the geometric properties of two very different lines –ellipse and eight-centered oval– can be quasi-equivalent (having barely distinguishable differences), therefore computations of the orbits can hide the real physics laws. By contrast and as an added value, this paper also shows to students, and interested people, that they can change an ellipse by a quasi-equivalent eight-centered oval, if they consider that their negligible geometric differences are assumable in their concrete problems.

This paper has academic nature, nevertheless it is an application from [9] that we think can be interesting to students and instructors of undergraduate level of physics. With low level of geometric and computational techniques the readers can reproduce the results of the present paper. The most important thing is that these results clearly show that it is not enough take numerical measures of the orbits of the bodies to decide which are the real equations and laws that govern them. Errors in measures, even if they are very small, can hide subtly the differences between different laws. Therefore in addition to taking measures, always depth studies of the physic which governs the motions of bodies are required. More specifically: here in this paper, we show the example of the planets' trajectory; because

It is well known that Johannes Kepler, in the 17th century, made a search for a better description of planetary motion and, in *Astronomia Nova* [10] (1609), he provided arguments for elliptical trajectory of the planets around the Sun. However, in 1675, Giovanni Domenico Cassini did not agree with Kepler and he tried to prove that the planetary orbits were Cassini's ovals [11]. Recently, Sivardiè [12] explored this question and concluded that the difference between the Kepler's ellipse and Cassini's oval is as distinguishable as that between the Kepler's ellipse and the circular orbit; therefore, if we discard the circle in favour of the ellipse, then, we also should discard the oval with the same argument. In the work [13], Morgado and Soares analyzed this possibility and they show that it is difficult to decide in favour of one